UNIT -1
LINEAR WAVE SHAPPING

Contents:

- High pass, Low pass circuits
- High pass and Low pass circuits response for:
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INTRODUCTION

Linear systems are those that satisfy both homogeneity and additivity.

(i) Homogeneity: Let *x* be the input to a linear system and *y* the corresponding output, as shown in Fig. 1.1. If the input is doubled (2*x*), then the output is also doubled (2*y*). In general, a system is said to exhibit homogeneity if, for the input *nx* to the system, the corresponding output is *ny* (where *n* is an integer). Thus, a linear system enables us to predict the output.

FIGURE 1.1 A linear system

(ii) Additivity: For two input signals x_1 and x_2 applied to a linear system, let y_1 and y_2 be the corresponding output signals. Further, if $(x_1 + x_2)$ is the input to the linear system and $(y_1 + y_2)$ the corresponding output, it means that the measured response will just be the sum of its responses to each of the inputs presented separately. This property is called additivity. Homogeneity and additivity, taken together, comprise the principle of superposition.

(iii) Shift invariance: Let an input *x* be applied to a linear system at time t_1. If the same input is applied at a different time instant t_2, the two outputs should be the same except for the corresponding shift in time. A linear system that exhibits this property is called a shift-invariant linear system. All linear systems are not necessarily shift invariant.

A circuit employing linear circuit components, namely, *R*, *L* and *C* can be termed a linear circuit. When a sinusoidal signal is applied to either *RC* or *RL* circuits, the shape of the signal is preserved at the output, with a change in only the amplitude and the phase. However, when a non-sinusoidal signal is transmitted through a linear network, the form of the output signal is altered. The process by which the shape of a non-sinusoidal signal passed through a linear network is altered is called linear wave shaping. We study the response of high-pass *RC* and *RL* circuits to different types of inputs in the following sections.

1. HIGH-PASS CIRCUITS

Figures 1.2(a) and 1.2(b) represent high-pass *RC* and *RL* circuits, respectively.

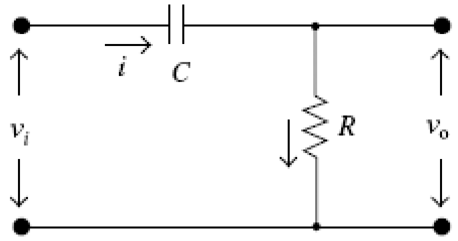


FIGURE 1.2(a) A high-pass RC circuit

Consider the high-pass *RC* circuit shown in Fig. 1.2(a). The capacitor offers a low reactance ($X_C = 1/j\omega C$) as the frequency increases; hence, the output is large. Consequently, high-frequency signals are passed to the output with negligible attenuation whereas, at low frequencies, due to the large reactance offered by the condenser, the output signal is small. Similarly, in the circuit shown in Fig. 1.2(b), the inductive reactance X_L ($= j\omega L$) increases with frequency, leading to a large output. At low frequencies, the reactance of the inductor X_L becomes small; hence, the output is small. Therefore, the circuits in Fig 1.2(a) and (b) are called high-pass circuits. In the case of *L*, X_L is directly proportional to frequency; and in the case of *C*, X_C is inversely proportional to frequency. *C* and *L* may therefore be called inverse circuit elements. Thus, in the high-pass circuit of Fig.

1.2(a), *C* appears as a series element; and in the high-pass circuit of Fig. 1.2(b), *L* appears as a shunt element. The time constant τ is given by: $\tau = RC = L/R$.

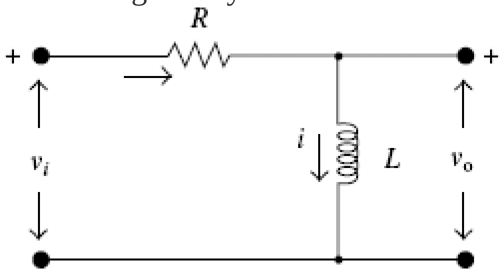


FIGURE 1.2(b) A high-pass *RL* circuit

What will be the response if different types of inputs such as sinusoidal, step, pulse, square wave, exponential and ramp are applied to a high-pass circuit?

Response of the High-pass *RC* Circuit to Sinusoidal Input

Let us consider the response of a high-pass *RC* circuit, shown in Fig. 1.2(a) when a sinusoidal signal is applied as the input. Here:

$$v_o = v_i \frac{R}{R + \frac{1}{j\omega C}} \qquad \text{------------- (1)}$$

$$\left|\frac{v_o}{v_i}\right| = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{R}{R\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}}$$

Let

$$\omega_1 = \frac{1}{CR} = \frac{1}{\tau} \qquad \text{------------------- (2)}$$

where, $\tau = RC$, the time constant of the circuit.

$$\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega_1}{\omega}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{T}{\tau}\right)^2}} \qquad (3)$$

The signal undergoes a phase change and the phase angle, θ, is given by:

$\theta = \tan^{-1}(\omega_1/\omega) = \tan^{-1}(T/\tau)$

At $\omega = \omega_1$:

$$\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{2}} = 0.707$$

Hence, f_1 is the lower half-power frequency of the high-pass circuit. The expression for the output for the circuits in Figs. 1.2(a) and (b) is the same as given by Eq. (3). Figure 1.3(a) shows a typical frequency–response curve for a sinusoidal input to a high-pass circuit. The frequency response and the phase shift of the circuit shown in Fig. 1.2(a) are plotted in Figs. 1.3(b) and 1.3(c), respectively, for different values of τ.

From Fig. 1.3(b), it is seen that as the half-power frequency decreases for larger values of τ, the gain curve shows a sharper rise. From Fig. 1.3(c), it is seen that if $T/\tau > 20$, the phase angle approaches approximately 90°.

Response of the High-pass *RC* Circuit to Step Input

A step voltage, shown in Fig.1.4(a), is represented mathematically as:

$$\left.\begin{aligned} v_i &= 0 \quad \text{for} \quad t < 0 \\ v_i &= V \quad \text{for} \quad t \geq 0 \end{aligned}\right\} \qquad (4)$$

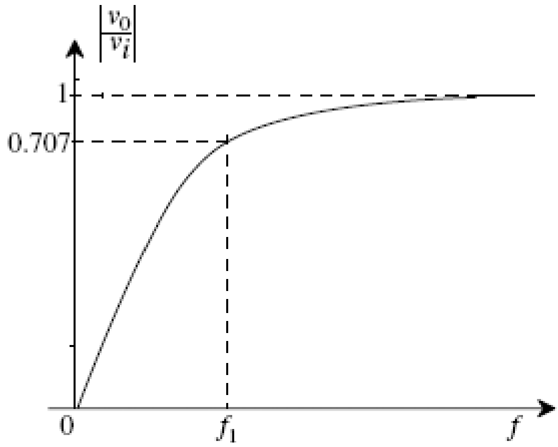


FIGURE 1.3(a) A typical frequency response curve for a sinusoidal input

A step is a sudden change in voltage, say at an instant t = 0, from say zero to V, in which case, it is called a positive step. The voltage change could also be from zero to -V, in which case it is called a negative step. This is an important signal in pulse and digital circuits. For instance, consider an *n–p–n* transistor in the CE mode. Assume that V_{BE} = 0. Then, the voltageat the collector is approximately V_{CC}. Now if a battery with voltage V_σ is connected so that $V_{BE} = V_\sigma$, as the device is switched ON, the voltage at the collector which earlier was V_{CC}, now falls to $V_{CE(\text{sat})}$. This means a negative step is generated at the collector. However, if the transistor is initially turned ON so that the voltage at its collector is $V_{CE(\text{sat})}$ and V_{BE} is made zero, then as the transistor is switched OFF, the voltage at its collector rises to V_{CC}. A positive step is now generated at its collector.

For a step input, let the output voltage be of the form:

$$v_o = B_1 + B_2 e^{-t/\tau} \qquad (5)$$

where, $\tau = RC$, the time constant of the circuit.

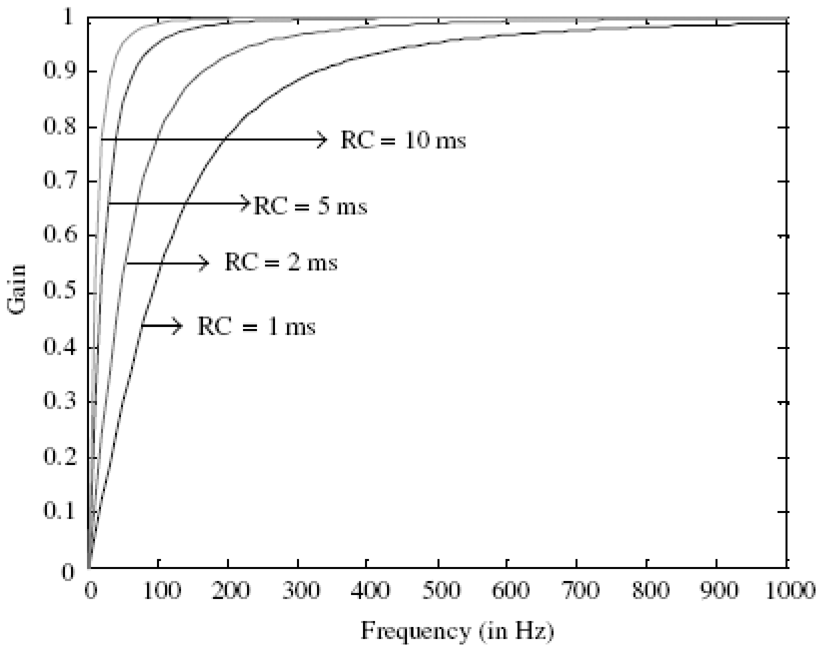


FIGURE 1.3(b) The frequency–response curve for different values of τ

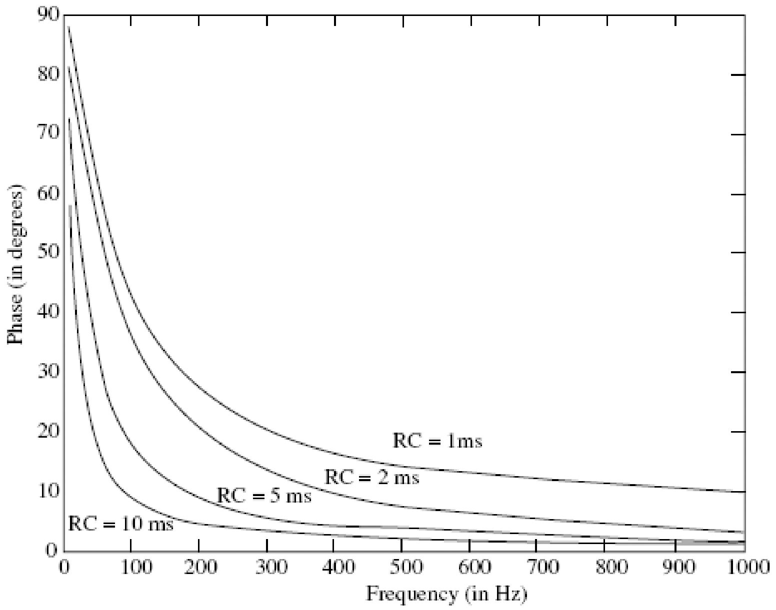


FIGURE 1.3(c) Phase versus frequency curve for different values of τ

B_1 is the steady-state value of the output voltage because as $t \to \infty$, $v_o \to B_1$.
Let the final value of this output voltage be called v_f. Then:

$$v_f = B_1 \qquad (6)$$

B_2 is determined by the initial output voltage. At $t = 0$, when the step voltage is applied, the change at the output is the same as the change at the input, because a capacitor is connected between the input and the output. Hence,

$$v_i = v_o = B_1 + B_2 \qquad (7)$$

Therefore,

$$B_2 = v_i - B_1$$

Using Eq. (6):

$$B_2 = v_i - v_f \qquad (8)$$

Substituting the values of B_1 and B_2 from Eqs. (6) and (8) respectively in Eq. (5), the general solution is given by the relation:

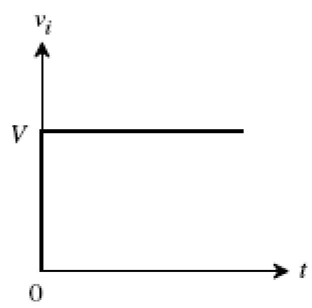
$$v_o = v_f + (v_i - v_f)\, e^{-t/\tau} \qquad (9)$$

For a high-pass RC circuit, let us calculate v_i and v_f. As the capacitor blocks the dc component of the input, $v_f = 0$. Since the capacitor does not allow sudden voltage changes, a change in the voltage of the input signal is necessarily accompanied by a corresponding change in the voltage of the output signal. Hence, at $t = 0+$ when the input abruptly rises to V, the output also changes by V.

Therefore, $v_i = V$.

Substituting the values of v_f and v_i in Eq. (9):

$$v_o(t) = Ve^{-t/\tau} \qquad (10)$$



$$x = \frac{t}{\tau} \quad (11)$$

$$\frac{v_o(t)}{V} = e^{-x} \quad (12)$$

$v_0(t)/V$ for x varying from 0 to 5 is shown in Table 1.1. The response of the circuit is plotted in Fig. 1.4(b).

At $t = 0$, when a step voltage V is applied as input to the high-pass circuit, as the capacitor will not allow any sudden changes in voltage, it behaves as a short circuit. Hence, the input voltage V appears at the output. As the input remains constant, the charge on the capacitor discharges exponentially with the time constant τ. After approximately 5τ, when τ is small, the output reaches the steady-state value. As τ becomes large, it takes a longer time for the charge on the capacitor to decay; hence, the output takes longer to reach the steady-state value. In general, the response of the circuit to different types of inputs is obtained by formulating the differential equation and solving for the output.

For the circuit in Fig. 1.2(a):

$$v_i = \frac{1}{C}\int i dt + v_o \quad (13)$$

But $v_o = iR$

$$i = \frac{v_o}{R} \quad (14)$$

$$\therefore v_i = \frac{1}{RC}\int v_o dt + v_o \quad (15)$$

For a step input, put $v_i = V$ and $RC = \tau$. Taking Laplace transforms:

$$\frac{V}{s} = \frac{v_o(s)}{s\tau} + v_o(s) \qquad v_o(s)\left(1 + \frac{1}{s\tau}\right) = \frac{V}{s} \qquad v_o(s)\left(s + \frac{1}{\tau}\right) = V$$

$$v_o(s) = \frac{V}{\left(s + \frac{1}{\tau}\right)} \quad (16)$$

Taking Laplace inverse:

$$v_o(t) = Ve^{-t/\tau} \quad (17)$$

Fall time (t_f): When a step voltage V is applied to a high-pass circuit, the output suddenly changes as the input and then the capacitor charges to V. Once the capacitor C is fully charged, it behaves as an open circuit for the dc input signal. Hence, in the steady-state, the output should be zero. However, the output does not reach this steady-state instantaneously; there is some time delay before the voltage on the capacitor decays and reaches the steady-state value. The time taken for the output voltage to fall from 90 per cent of its initial value to 10 per cent of its initial value is called the fall time. It indicates how fast the output reaches its steady-state value.

The output voltage at any instant, in the high-pass circuit is given by Eq. (17). At $t = t_1$, $v_o(t_1)$ = 90% of V = 0.9 V. Therefore,

$0.9 = e^{-t_1/\tau}$ $e^{t1/\tau} = 1/0.9 = 1.11$ $t_1/\tau = \ln(1.11)$

$t_1 = \tau \ln(1.11) = 0.1\tau$

At $t = t_2$, $v_o(t) = 10\%$ of $V = 0.1\ V$. Hence,

$0.1 = e^{-t_2/\tau}$ $e^{t2/\tau} = 1/0.1 = 10$ $t_2 = \tau \ln(10) = 2.3\tau$

The fall time is calculated as:

$$t_f = t_2 - t_1 = 2.3\tau - 0.1\tau = 2.2\tau \quad (18)$$

The lower half-power frequency of the high-pass circuit is:

$$f_1 = \frac{1}{2\pi RC} \quad \text{--------------------------------} \quad (19)$$

$$\tau = RC = \frac{1}{2\pi f_1} \quad \text{--------------------------------} \quad (20)$$

$$t_f = 2.2\tau = \frac{2.2}{2\pi f_1} = \frac{0.35}{f_1} \quad \text{--------------------------------} \quad (21)$$

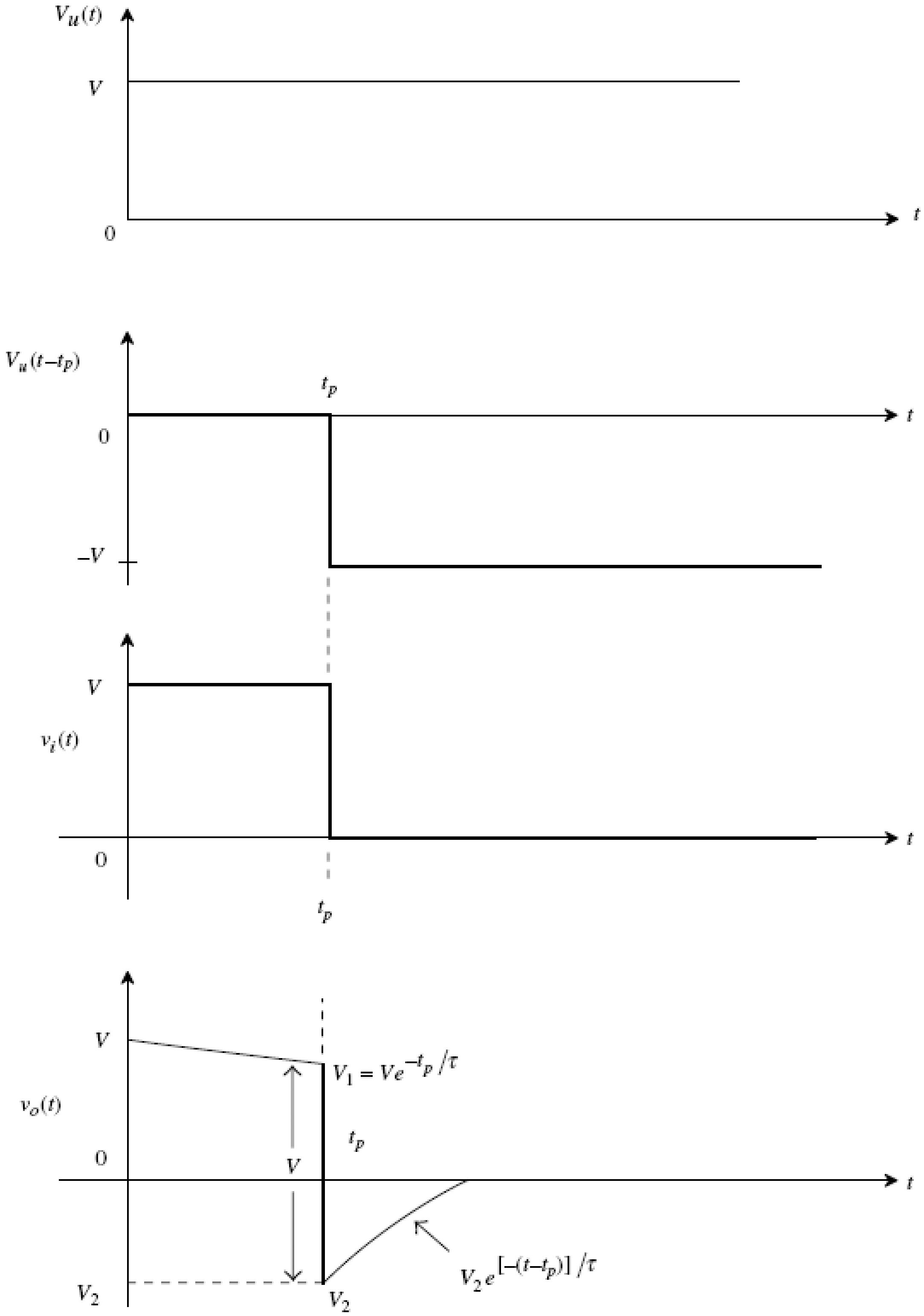
Hence, the fall time is inversely proportional to f_1, the lower half-power frequency. As f_1, is inversely proportional to τ, the shape of the signal at the output changes with τ.

Response of the High-pass *RC* Circuit to Pulse Input

A positive pulse is mathematically represented as the combination of a positive step followed by a delayed negative step i.e., $v_i = V_u(t) - V_u(t - t_p)$ where, t_p is the duration of the pulse as shown in Fig. 1.6.

To understand the response of a high-pass circuit to this pulse input, let us trace the sequence of events following the application of the input signal.

At $t = 0$, v_i abruptly rises to V. As a capacitor is connected between the input and output, the output also changes abruptly by the same amount. As the input remains constant, the output decays exponentially to V_1 at $t = t_p$. Therefore,



$V_u(t)$
V
0
t
$V_u(t-t_p)$
t_p
0
$-V$
t
V
$v_i(t)$
0
t
t_p
V
$v_o(t)$
$V_1 = Ve^{-t_p/\tau}$
t_p
0
V
t
$V_2 e^{[-(t-t_p)]/\tau}$
V_2
V_2

FIGURE 1.6 Pulse input and output of a high-pass circuit

$$V_1 = Ve^{-t_p/\tau} \quad (22)$$

At $t = t_p$, the input abruptly falls by V, v_o also falls by the same amount. In other words, $v_o = V_1 - V$. Since V_1 is less than V; v_o is negative and its value is V_2 and this decays to zero exponentially. For $t > t_p$,

$$v_o = V_2\, e^{[-(t-t_p)]/\tau} = (V_1 - V)\, e^{[-(t-t_p)]/\tau} \quad (23)$$

Substituting Eq. (22) in Eq. (23):

$$v_o = V(e^{-t_p/\tau} - 1)\, e^{[-(t-t_p)]/\tau} \quad (24)$$

The response of high-pass circuits with different values of τ to pulse input is plotted in Fig. 1.7. As is evident from the preceding discussion, when a pulse is passed through a high-pass circuit, it gets distorted. Only when the time constant τ is very large, the shape of the pulse at the output is preserved, as can be seen from Fig. 1.7(b). However, as shown in Fig. 1.7(c), when the time constant τ is neither too small nor too large, there is a tilt (also called a sag) at the top of the pulse and an under-shoot at the end of the pulse. If $\tau << t_p$, as in Fig. 1.7(d), the output comprises a positive spike at the beginning of the pulse and a negative spike at the end of the pulse. In other words, a high-pass circuit converts a pulse into spikes by employing a small time constant; this process is called peaking.

If the distortion is to be negligible, τ has to be significantly larger than the duration of the pulse. In general, there is an undershoot at the end of the pulse. The larger the tilt (for small τ), the larger the undershoot and the smaller the time taken for this undershoot to decay to zero. The area above the reference level (A_1) is the same as the area below the reference level (A_2). Let us verify this using Fig. 1.8.

Area A_1: For $0 < t < t_p$:

$v_o = Ve^{-t/\tau}$

$$A_1 = \int_0^{t_p} Ve^{-t/\tau} dt = \left[-V\tau e^{-t/\tau}\right]_0^{t_p}$$

Similarly,

$$A_1 = \left[-V\tau e^{-t_p/\tau} + V\tau\right] = V\tau(1 - e^{-t_p/\tau}) \quad (25)$$

$$A_2 = \int_{t_p}^{\infty} V(e^{-t_p/\tau} - 1)e^{-(t-t_p)/\tau}\,dt = \int_{t_p}^{\infty} \left[Ve^{-t/\tau} - Ve^{-(t-t_p)/\tau}\right] dt$$

$$= \left[\frac{Ve^{-t/\tau}}{\frac{-1}{\tau}}\right]_{t_p}^{\infty} - \left[V\frac{1}{\frac{-1}{\tau}}e^{-(t-t_p)/\tau}\right]_{t_p}^{\infty}$$

$$A_2 = \left[V\tau e^{-t_p/\tau} - V\tau\right] = -V\tau(1 - e^{-t_p/\tau}) \qquad (26)$$

From Eqs. (25) and (26) it is evident that

$$|A_1| = |A_2| \qquad (27)$$

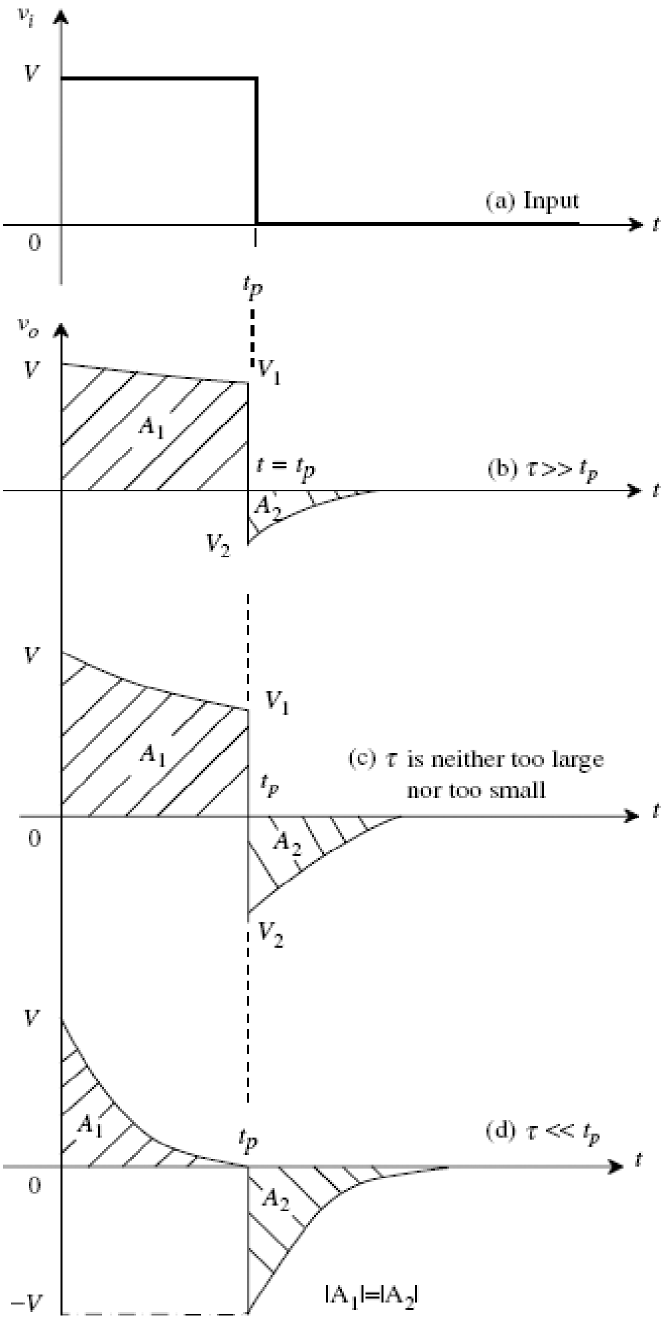


FIGURE 1.7 The response of a high-pass circuit to a pulse input

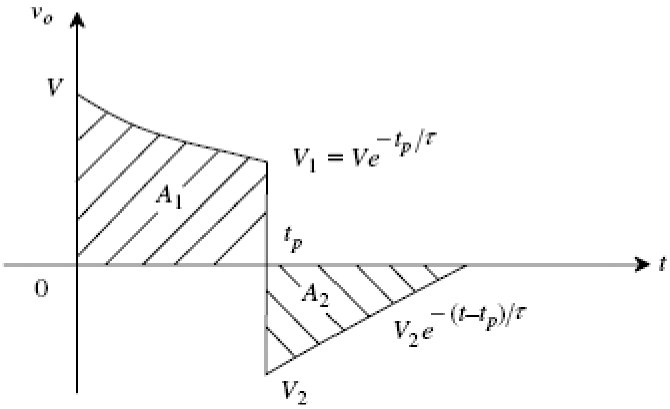


FIGURE 1.8 The calculation of A_1 and A_2

EXAMPLE

Example 1: A pulse of amplitude 10 V and duration 10 μs is applied to a high-pass *RC* circuit. Sketch the output waveform indicating the voltage levels for (i) $RC = t_p$, (ii) $RC = 0.5t_p$ and (iii) $RC = 2t_p$.

Solution:

1. When $RC = t_p = \tau$
   At $t = t_p$

$V_1 = 10\ e^{-(10\times10-6)/(10\times10-6)} = 10^{-1} = 3.678$ V

$v_{o1} = 10\ e^{-t/(10\times10-6)}$ for $t < t_p$

$v_o(t > t_p) = (V_1 - 10)e^{-(t-10\times10-6)/(10\times10-6)} = -6.322\ e^{-(t-10\times10-6)/(10\times10-6)}$

2. When $RC = \tau = 0.5t_p$
   At $t = t_p$

$V_1 = 10\ e^{-(10\times10-6)/(0.5\times10\times10-6)} = 10e^{-2} = 1.35$ V

$v_{o1} = 10\ e^{-t/(0.5\times10\times10-6)}$ for $t < t_p$

$v_o(t > t_p) = -8.65\ e^{-(t-10\times10-6)/(0.5\times10\times10-6)}$

3. When $RC = \tau = 2t_p$
   At $t = t_p$

$V_1 = 10\ e(^{-10\times10-6)/(2\times10\times10-6)} = 10e^{-0.5} = 6.05$ V

$v_{o1} = 10\ e^{-t/(2\times10\times10-6)}$ for $t < t_p$

$v_o(t > t_p) = -3.935\ e^{-(t-10\times10-6)/(2\times10\times10-6)}$

Based on these results, the output waveforms are sketched as in Fig. 1.9(a), (b) and (c) corresponding to cases (i), (ii) and (iii), respectively.

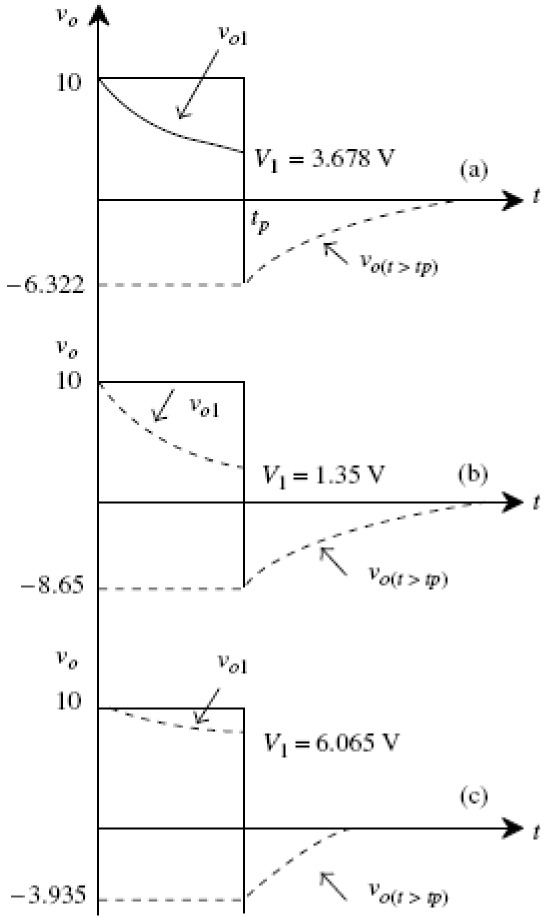


FIGURE 1.9(b) The response of a high-pass circuit for different values of τ

From Example 2.2, it is seen that when τ is large, the amplitude distortion in the output is minimal, i.e., the shape of the signal is almost preserved in the output. As the value of τ decreases, the charge on the capacitor decreases by a larger amount during the period the input remains constant. Consequently, the output is distorted. If τ decreases still further, it can be seen that the output contains positive and negative spikes. The shape of the signal in the output is essentially decided by the time constant of the circuit.

Response of the High-pass *RC* Circuit to Square-wave Input

A waveform that has a constant amplitude, say, V' for a time T_1 and has another constant amplitude, V'' for a time T_2, and which is repetitive with a time $T = (T_1 + T_2)$, is called a square wave. In a symmetric square wave, $T_1 = T_2 = T/2$. Figure 1.10 shows typical input–output waveforms of the high-pass circuit when a square wave is applied as the input signal.

As the capacitor blocks the DC, the DC component in the output is zero. Thus, as expected, even if the signal at the input is referenced to an arbitrary dc level, the output is always referenced to the zero level. It can be proved that whatever the dc component associated with a periodic input waveform, the dc level of the steady-state output signal for the high-pass circuit is always zero as shown in Fig. 1.10. To verify this statement, we write the KVL equation for the high-pass circuit:

$$v_i = \frac{q}{C} + v_o \qquad (28)$$

where, q is the charge on the capacitor. Differentiating with respect to t:

$$\frac{dv_i}{dt} = \frac{1}{C}\frac{dq}{dt} + \frac{dv_o}{dt} \qquad (29)$$

But $i = \frac{dq}{dt}$

Substituting this condition in Eq. (29):

$$\frac{dv_i}{dt} = \frac{i}{C} + \frac{dv_o}{dt}$$

Since $v_o = iR$, $i = v_o/R$ and $RC = \tau$. Therefore,

$$\therefore \frac{dv_i}{dt} = \frac{v_o}{\tau} + \frac{dv_o}{dt} \qquad (30)$$

Multiplying by dt and integrating over the time period T we get:

$$\int_0^T dv_i = [v_i]_0^T = v_i(T) - v_i(0) \qquad (31)$$

$$\int_0^T \frac{v_o}{\tau}dt = \frac{1}{\tau}\int_0^T v_o dt \qquad (32)$$

$$\int_0^T dv_o = [v_o]_0^T = v_o(T) - v_o(0) \qquad (33)$$

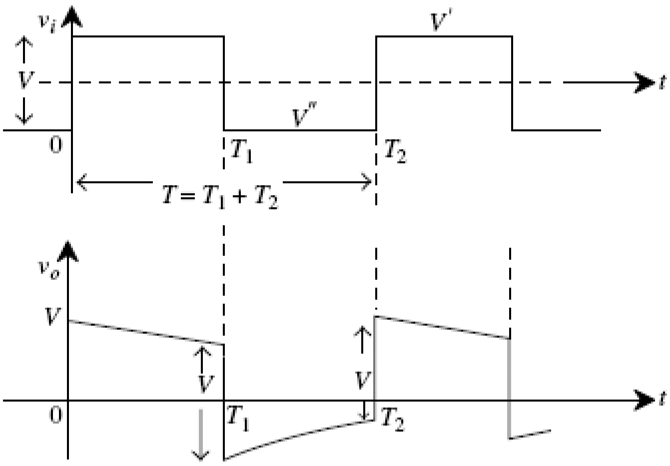


FIGURE 1.10 A typical steady-state output of a high-pass circuit with a square wave as input

From Eqs. (30), (31), (32) and (33):

$$v_i(T) - v_i(0) = \frac{1}{\tau}\int_0^T v_o dt + [v_o(T) - v_o(0)] \quad (34)$$

Under steady-state conditions, the output and the input waveforms are repetitive with a time period T. Therefore, $v_i(T) = v_o(T)$ and $v_i(0) = v_o(0)$. Hence, from Eq. (34):

$$\int_0^T v_o dt = 0 \quad (35)$$

As the area under the output waveform over one cycle represents the DC component in the output, from Eq. (35) it is evident that the DC component in the steady-state is always zero. Now let us consider the response of the high-pass *RC* circuit for a square-wave input for different values of the time constant τ, as shown in Fig. 1.11.

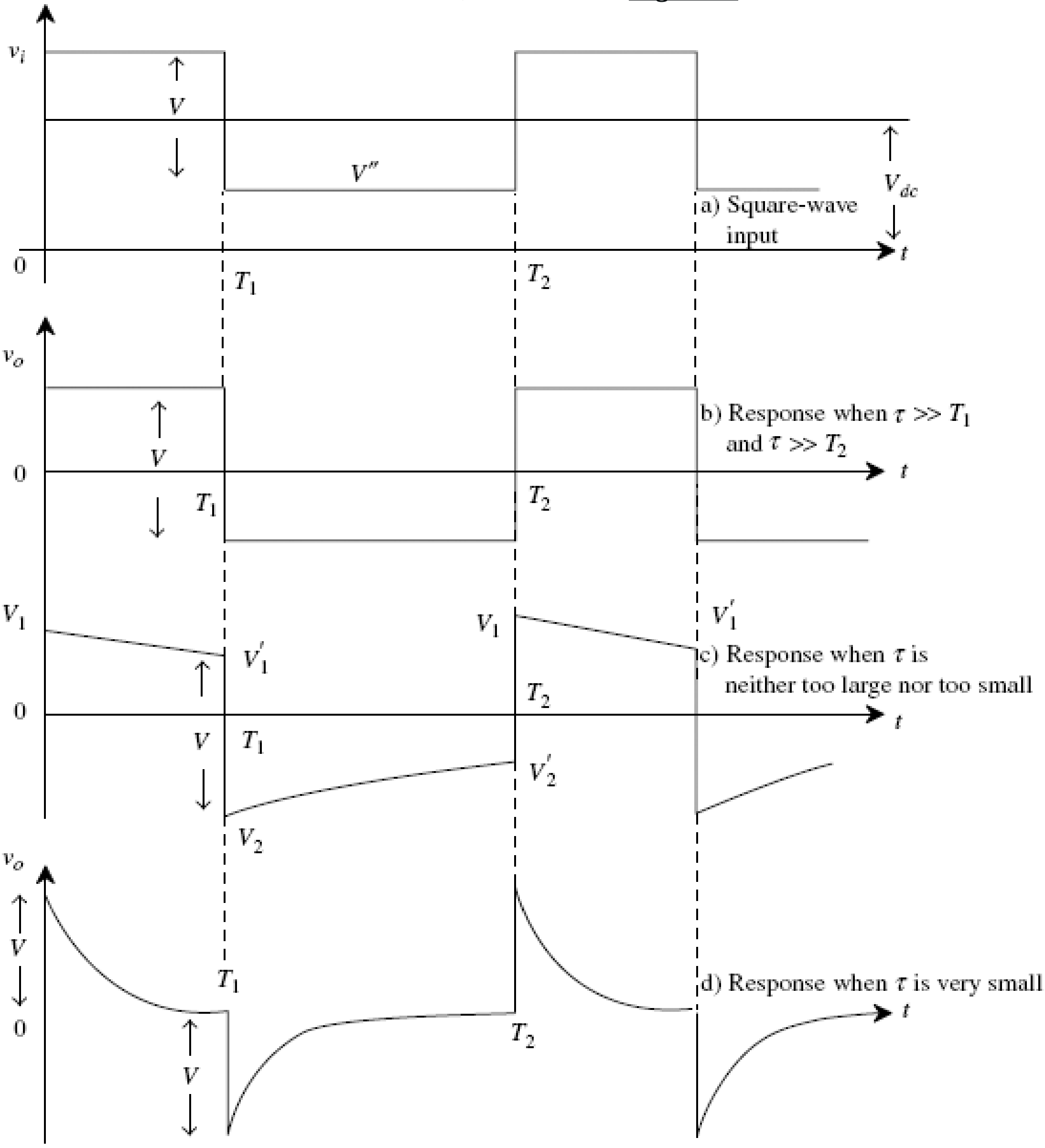


FIGURE 1.11 The response of a high-pass circuit for a square-wave input

As is evident from the waveform in Fig. 1.11(b), there is no appreciable distortion in the output if τ is large. The output is almost the same as the input except for the fact that there is no DC component in the output. As τ decreases, as in Fig. 1.11(c), there is a tilt in the positive duration (amplitude decreases from V_1 to V_1' during the period 0 to T_1) and there is also a tilt in the negative duration (amplitude increases from V_2 to V_2' during the period T_1 to T_2). A further decrease in the value of τ [see Fig. 1.11(d)] gives rise to positive and negative spikes. There is absolutely no resemblance between the signals at the input and the output. However, this condition is imposed on high-pass circuits to derive spikes. In case a pulse is required to trigger another circuit, we see that the pulses obtained either at the rising edge (positive spike) or at the trailing edge (negative spike) may be used to edge trigger a flip-flop, as discussed in later chapters in the book. Let us consider the typical response of the high-pass circuit for a square-wave input shown in Fig. 1.12.

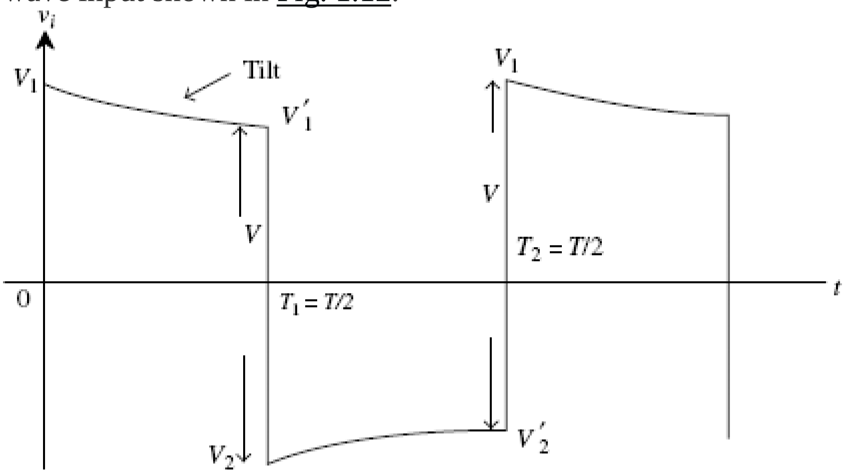


FIGURE 1.12 The typical response of a high-pass *RC* circuit for a square-wave input

From Fig. 1.12 and using Eq. (17) we have:

$$V_1' = V_1 e^{-T_1/\tau} \quad \text{and} \quad V_1' - V_2 = V$$
$$V_2' = V_2 e^{-T_2/\tau} \quad \text{and} \quad V_1 - V_2' = V \qquad (36)$$

For a symmetric square wave $T_1 = T_2 = T/2$. And, because of symmetry:

$$V_1 = -V_2 \quad \text{and} \quad V_1' = -V_2' \qquad (37)$$

From Eq. (36):

$$V_1' - V_2 = V$$

But

$$V_1' = V_1 e^{-T_1/\tau}$$

Therefore,

$$V_1 e^{-T_1/\tau} - V_2 = V \quad (38)$$

From Eq. (37):

$V_1 = -V_2$

Substituting in Eq. (38):

$V_1 e^{-T_1/\tau} + V_1 = V \qquad V_1(1 + e^{-T_1/\tau}) = V$

Thus,

$$V_1 = \frac{V}{1 + e^{-T_1/\tau}} \quad (39)$$

For a symmetric square wave, as $T_1 = T_2 = T/2$, Eq. (2.39) is written as:

$$V_1 = \frac{V}{1 + e^{-T/2\tau}} \quad (40)$$

But

$$V_1' = V_1 e^{-T/2\tau} = \frac{V}{(1 + e^{-T/2\tau})} \times e^{-T/2\tau} \quad (41)$$

There is a tilt in the output waveform. The percentage tilt, P, is defined as:

$$P = \frac{V_1 - V_1'}{\frac{V}{2}} \times 100\% \quad (42)$$

$$P = \frac{\frac{V}{1 + e^{-T/2\tau}} - \frac{V e^{-T/2\tau}}{1 + e^{-T/2\tau}}}{\frac{V}{2}} \times 100\,\% = \frac{V}{1 + e^{-T/2\tau}} \frac{\left[1 - e^{-T/2\tau}\right]}{\frac{V}{2}} \times 100\,\%$$

$$P = \frac{\left(1 - e^{-T/2\tau}\right)}{\left(1 + e^{-T/2\tau}\right)} \times 200\% \quad (43)$$

If $T/2\tau << 1$,

$$e^{-T/2\tau} = 1 - \frac{T}{2\tau} \quad (44)$$

Therefore,

$$P = \frac{1 - \left(1 - \frac{T}{2\tau}\right)}{1 + \left(1 - \frac{T}{2\tau}\right)} \times 200\% = \frac{\frac{T}{2\tau}}{2 - \frac{T}{2\tau}} \times 200\% \cong \frac{T}{2\tau} \times 100\% \text{ since } \frac{T}{2\tau} << 1$$

Thus, for a symmetrical square wave:

$$P = \frac{T}{2\tau} \times 100\%, \qquad (45)$$

Equation (45) tells us that the smaller the value of τ when compared to the half-period of the square wave ($T/2$), the larger is the value of P. In other words, distortion is large with small τ and is small with large τ. The lower half-power frequency, $f_1 = 1/2\pi\tau$.

Therefore,

$$\frac{1}{2\tau} = \pi f_1$$

Putting in Eq. (45)

$$P = \pi f_1 T \times 100\%$$

Therefore,

$$P = \frac{\pi f_1}{f} \times 100\% \qquad \text{since } T = \frac{1}{f}. \qquad (46)$$

Let us calculate and plot the response by taking specific examples.

EXAMPLE

Example 3: A 10 Hz square wave whose peak-to-peak amplitude is 2 V is fed to an amplifier. Calculate and plot the output waveform if the lower 3-dB frequency is 0.3 Hz.

Solution: Let C be the condenser through which the signal is connected to the amplifier, having an input resistance R, as shown in Fig. 1.13(a). This is essentially the high-pass circuit in Fig. 1.1(a).

The lower 3-dB frequency $f_1 = 0.3$ Hz

Input frequency is $f = 10$ Hz

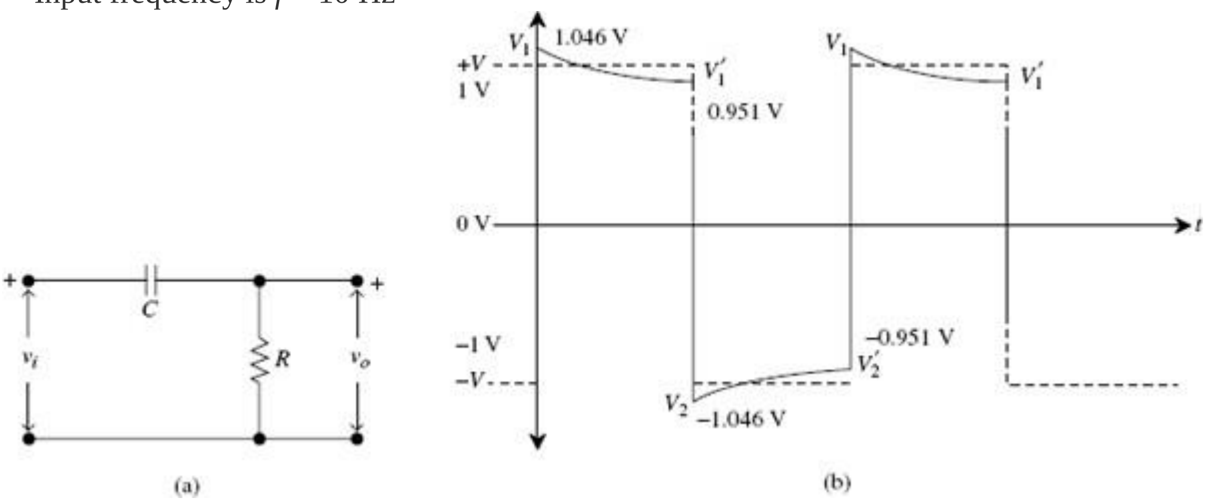


FIGURE 1.13(a) The coupling network and (b) The response of the circuit

$$\tau = RC = \frac{1}{2\pi f_1} = \frac{1}{2\pi(0.3)} = 0.53\text{ s}$$

$$T = \frac{1}{f} = \frac{1}{10} = 0.1\text{ s}$$

Therefore,

$$\frac{T}{2} = 0.05\text{ s}$$

$$V_1 = \frac{V}{1 + e^{-T/2\tau}} = \frac{2}{1 + e^{-0.05/0.53}} = 1.046\text{ V} \qquad V_1' = V_1 e^{-T/2\tau} = 1.046e^{-0.05/0.53} = 0.951\text{ V}$$

$$V_1 = -V_2 \quad \text{and} \quad V_1' = -V_2'$$

$$V_1 = |V_2| = 1.046\text{ V} \qquad V_1' = |V_2'| = 0.951\text{ V}$$

The response of the circuit is shown in Fig. 1.13(b).

EXAMPLE

Example 4: A 20-Hz symmetrical square wave, shown in Fig. 1.14(a), with peak-to-peak amplitude of 2 V is impressed on a high-pass circuit shown in Fig. 1.1(a) whose lower 3-dB frequency is 10 Hz. Calculate and sketch the output waveform. What is the peak-to-peak output amplitude of the above waveform?

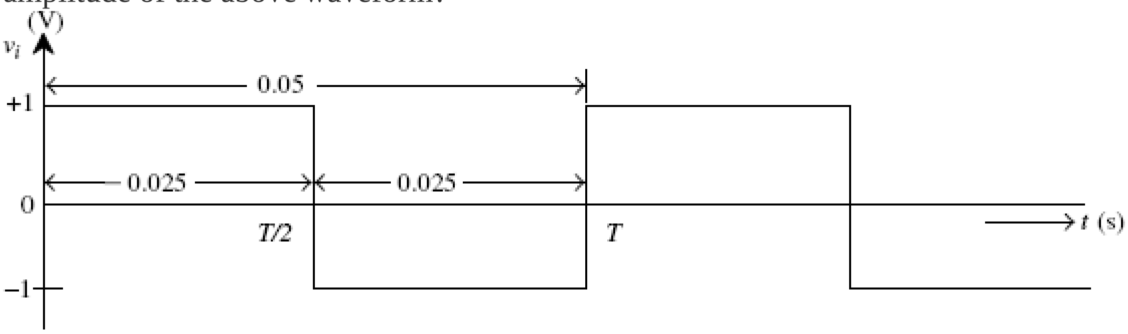


FIGURE 1.14(a) Input to the high-pass circuit

Solution: The lower 3-dB frequency:

$$f_1 = \frac{1}{2\pi RC} = 10\text{ Hz} \qquad RC = \tau = \frac{1}{2\pi f_1} = \frac{1}{2\pi \times 10} = 0.0159\text{ s}$$

Input signal frequency $f = 20$ Hz

Time period of the input $T = \frac{1}{f} = \frac{1}{20} = 0.05\text{ s}$

$$\frac{T}{2} = \frac{0.05}{2} = 0.025\text{ s}$$

Therefore, τ is small compared to $T/2$; so the capacitor charges and discharges appreciably in each half-cycle. Since the input is a symmetrical square wave, $V_1 = -V_2$, i.e., $|V_1| = |V_2|$, $V_1' = -V_2'$ i.e., $|V_1'| = |V_2'|$. The peak-to-peak input = 2 V. Hence,

$$V_1 = \frac{V}{1 + e^{(-T/2)/\tau}} = \frac{2}{1 + e^{-0.025/0.0159}} = 1.656\ \text{V} \qquad V_2 = -V_1 = -1.656\ \text{V}$$

Peak-to-peak value of output = $V_1 - V_2$ = 3.312 V.

$V_1' = V_1 e^{(-T/2)/\tau} = 1.656\ e^{-(0.025/0.0159)} = 0.344\ \text{V}$

$$V_1' = -V_2' = 0.344\ \text{V}$$

The output is plotted in Fig. 1.14(b).

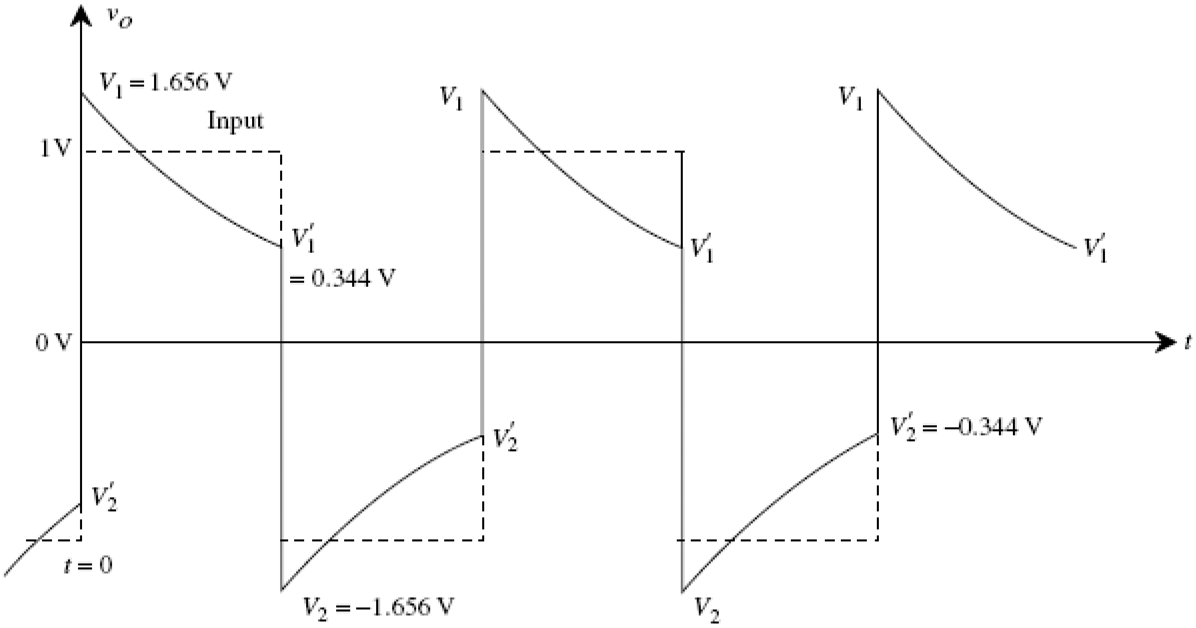


FIGURE 1.14(b) Output of the high-pass circuit for the given input

FIGURE 1.15 The output of the high-pass circuit for the specified input

Response of the High-pass *RC* Circuit to Exponential Input

When a pulse is applied as input to an amplifier, it may while appearing at the actual input terminals of the amplifier, have a finite rise time. The result is that the input to the amplifier is no longer a pulse with sharp rising edge, but an exponential. We would now like to know the response of the high-pass circuit to this exponential input. If the input to the high-pass circuit in Fig.1.2(a) is an exponential of the form:

$$v_i = V(1 - e^{-t/\tau_1}), \quad (47)$$

where, τ_1 is the time constant of the circuit that has generated the exponential signal as shown in Fig. 1.16(a).

From Eq. (30), we know:

$$\frac{dv_i}{dt} = \frac{v_o}{\tau} + \frac{dv_o}{dt}$$

As $v_i = V(1 - e^{-t/\tau 1})$,

$$\frac{dv_i}{dt} = \frac{V}{\tau_1} e^{-t/\tau_1} \quad (48)$$

Substituting Eq. (2.48) in Eq. (2.30):

$$\frac{V}{\tau_1} e^{-t/\tau_1} = \frac{v_o}{\tau} + \frac{dv_o}{dt} \quad (49)$$

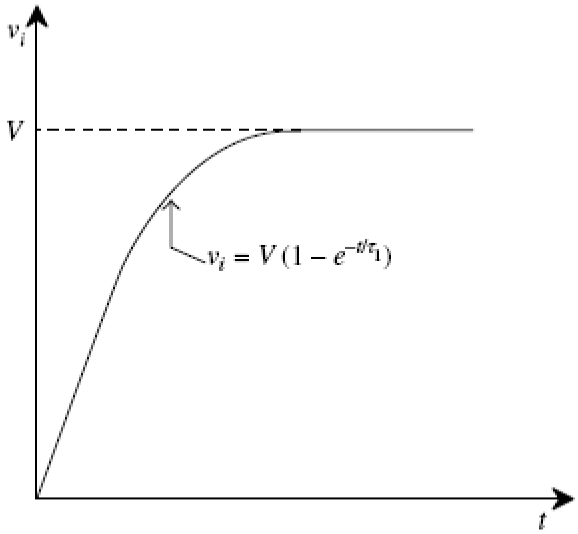


FIGURE 1.16(a) Exponential input

Taking Laplace transforms:

$$\frac{\frac{V}{\tau_1}}{\left(s + \frac{1}{\tau_1}\right)} = \frac{v_o(s)}{\tau} + s v_o(s)$$

where, τ is the time constant of the high-pass circuit.

$$\frac{\frac{V}{\tau_1}}{\left(s + \frac{1}{\tau_1}\right)} = v_o(s)\left(s + \frac{1}{\tau}\right)$$

Therefore,

$$v_o(s) = \frac{\frac{V}{\tau_1}}{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau}\right)} \quad (50)$$

Case 1: $\tau = \tau_1$

Applying partial fractions, Eq. (50) can be written as:

$$v_o(s) = \frac{A}{\left(s+\frac{1}{\tau_1}\right)} + \frac{B}{\left(s+\frac{1}{\tau}\right)} = \frac{\frac{V}{\tau_1}}{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau}\right)} \quad (51)$$

Therefore,

$$\frac{V}{\tau_1} = A\left(s+\frac{1}{\tau}\right) + B\left(s+\frac{1}{\tau_1}\right) \quad (52)$$

Put $s = -1/\tau_1$ in Eq. (52).

$$\frac{V}{\tau_1} = A\left(\frac{-1}{\tau_1}+\frac{1}{\tau}\right) \quad \text{or} \quad A = \frac{\frac{V}{\tau_1}}{\left(\frac{1}{\tau}-\frac{1}{\tau_1}\right)} = \frac{V}{\left(\frac{\tau_1}{\tau}-1\right)}$$

Now put $s = -1/\tau$ in Eq. (52). Then:

$$\frac{V}{\tau_1} = B\left(\frac{1}{\tau_1}-\frac{1}{\tau}\right)$$

Therefore,

$$B = \frac{-V}{\left(\frac{\tau_1}{\tau}-1\right)} \quad (53)$$

Substituting the values of A and B in Eq. (51):

$$v_o(s) = \frac{V}{\left(\frac{\tau_1}{\tau}-1\right)\left(s+\frac{1}{\tau_1}\right)} - \frac{V}{\left(\frac{\tau_1}{\tau}-1\right)\left(s+\frac{1}{\tau}\right)} = \frac{V}{\left(\frac{\tau_1}{\tau}-1\right)}\left[\frac{1}{\left(s+\frac{1}{\tau_1}\right)} - \frac{1}{\left(s+\frac{1}{\tau}\right)}\right]$$

Taking inverse Laplace transform:

$$v_o(t) = \frac{V}{\left(\frac{\tau_1}{\tau}-1\right)}\left(e^{-t/\tau_1} - e^{-t/\tau}\right) \quad (54)$$

This is the expression for the output voltage where $\tau \neq \tau_1$.

Let $t/\tau_1 = x$ and $\tau/\tau_1 = n$. For $n \neq 1$, i.e., $\tau \neq \tau_1$, we have from Eq. (54):

$$v_o(t) = \frac{V}{\left(\frac{1}{n} - 1\right)}\left(e^{-x} - e^{-x/n}\right) \quad \text{since } \frac{t}{\tau_1} \times \frac{\tau_1}{\tau} = \frac{x}{n} = \frac{t}{\tau}$$

Therefore,

$$v_o(t) = \frac{Vn}{(1-n)}\left(e^{-x} - e^{-x/n}\right) = \frac{Vn}{(n-1)}\left(e^{-x/n} - e^{-x}\right) \qquad (55)$$

If $\tau >> \tau_1$, the second term in the Eq. (2.55) is small when compared to the first. Thus,

$$v_o(t) \cong \frac{Vn}{(n-1)}e^{-x/n} = \frac{Vn}{(n-1)}e^{-t/\tau} \qquad (56)$$

Case 2: $\tau = \tau_1$, that is, $n = 1$.

$$v_o(s) = \frac{\frac{V}{\tau}}{\left(s + \frac{1}{\tau}\right)\left(s + \frac{1}{\tau}\right)} = \frac{\frac{V}{\tau}}{\left(s + \frac{1}{\tau}\right)^2}$$

Taking Laplace inverse:

$$v_o(t) = \frac{V}{\tau}te^{-t/\tau} \qquad (57)$$

As $t/\tau = x = t/\tau_1$ and $\tau/\tau_1 = n = 1$:

$$v_o(t) = Vxe^{-x} \qquad (58)$$

The response of the circuit is plotted for different values of n in Fig. 1.16(b).

From the response in Fig.1.16(b), it is seen that near the origin the output follows the input. Also, the smaller the value of n (= τ/τ_1 is small), the smaller is the output peak and the shorter is the duration of the pulse. As n increases, the peak becomes larger and the duration of the pulse becomes longer. Hence, the choice of n is based on the amplitude and duration of the pulse required for a specific application. The maximum output occurs when (dv_o/dt) = 0. From Eq. (55):

$$\frac{d}{dt}\left[V\frac{n}{n-1}(e^{-x/n} - e^{-x})\right] = 0$$

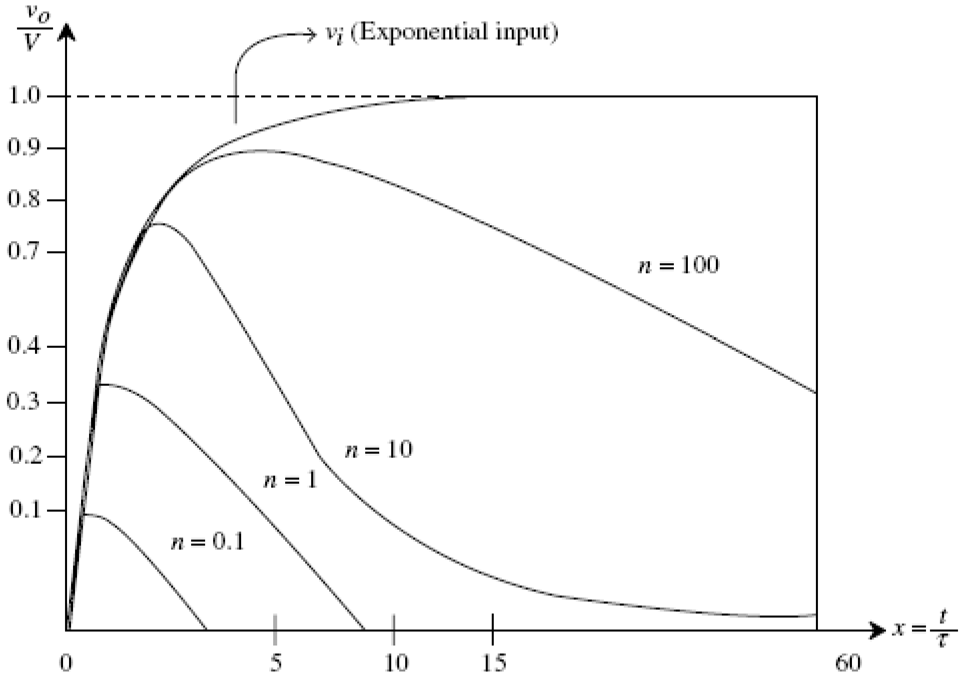


FIGURE 1.16(b) The response of a high-pass circuit to an exponential input

$$V\frac{n}{n-1}\left[\left(\frac{-1}{n}\right)\left(\frac{1}{\tau}\right)e^{-x/n} - e^{-x}\left(\frac{-1}{\tau}\right)\right] = 0$$

$$V\frac{n}{n-1}\left[\left(\frac{e^{-x}}{\tau} - \frac{e^{-x/n}}{n\tau}\right)\right] = 0 \qquad e^{-x} = \frac{e^{-x/n}}{n}$$

$$n = e^{x\left(1-\frac{1}{n}\right)} = e^{\left(\frac{x(n-1)}{n}\right)}$$

$$\ln n = \frac{x(n-1)}{n}$$

$$x = \frac{n}{n-1}\ln n \qquad (59)$$

Since $x = t/\tau$, from Eq. (59), the time taken to rise to the peak t_p is given by:

$$t_p = \tau\frac{n}{n-1}\ln n$$

From Eq. (59):

$$-x = \frac{-n}{n-1}\ln n = \ln\left[n^{n/(1-n)}\right] \qquad (60)$$

To obtain the maximum value of the output, substitute this value of $-x$ from Eq. (60) in the expression for $v_o(t)$ in Eq. (55).

$$v_o(\max) = \frac{Vn}{n-1}\exp\left[\frac{1}{n}\ln n^{[n/(1-n)]} - \ln n^{[n/(1-n)]}\right]$$

$$= V\frac{n}{n-1}\exp\left[\ln n^{[1/1-n]} - \ln n^{[n/(1-n)]}\right]$$

$$= V\frac{n}{n-1}\left[n^{[1/(1-n)]} - n^{[n/(1-n)]}\right] = \frac{V}{n-1}\left[n^{1+[1/(1-n)]} - n^{1+[n/(1-n)]}\right]$$

$$= \frac{V}{n-1}\left[n^{(2-n)/(1-n)} - n^{1/(1-n)}\right] = \frac{V}{n-1}\left[(n-1)^{1/(1-n)}\right] = Vn^{1/(1-n)} \quad \text{for } n \neq 1$$

$$\frac{v_o(\max)}{V} = n^{1/(1-n)} \quad \text{for } n \neq 1 \tag{61}$$

From the waveforms in Fig. 1.16(b) and the subsequent mathematical relations derived, it is seen that, if an exponential signal is applied as an input to a high-pass circuit, the output is a pulse whose duration depends on $n(= \tau/\tau_1)$, where τ_1 is the time constant of the previous circuit that has generated the exponential signal and τ is the time constant of the high-pass circuit under consideration. The smaller the value of n, the smaller the duration of this output pulse and also the smaller its amplitude. As n increases, the duration as well as the amplitude of this output pulse increases. Hence, depending on our requirement, we adjust the value of n.

it for the given exponential input when $\tau = \tau_1$

The input and the output waveforms are as shown in Fig. 1.17.

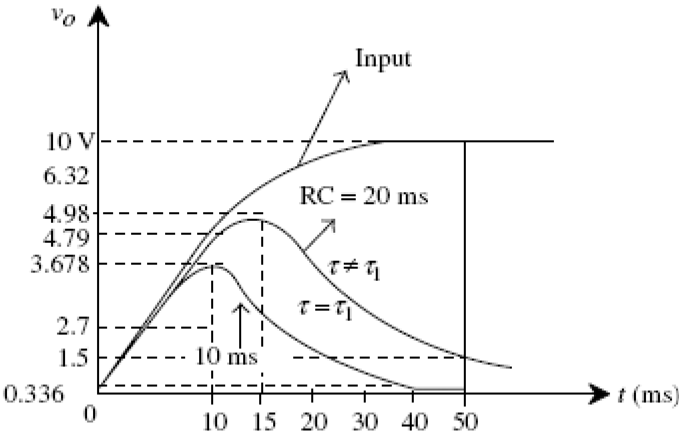


FIGURE 1.17 The input to and the output of the high-pass circuit

Response of the High-pass *RC* Circuit to Ramp Input

Ramp is a waveform in which the voltage increases linearly with time, for $t > 0$, and is zero for $t < 0$. It is used to move the spot in a CRO linearly with time along the x-axis. This type of waveform is generated by sweep circuits which we shall study later. However, if a ramp is applied as an input to a high-pass circuit, there could be deviation from linearity in the ouput. We can calculate and plot the ouput for different values of τ to understand how it influences the output. Let the input to the high-pass circuit be $v_i = \alpha t$ where, α is the slope, as shown in Fig. 1.18(a).

For the high-pass circuit, we have:

$$v_i = \frac{1}{\tau}\int v_o dt + v_o$$

$$\alpha t = \frac{1}{\tau}\int v_o dt + v_o \qquad (62)$$

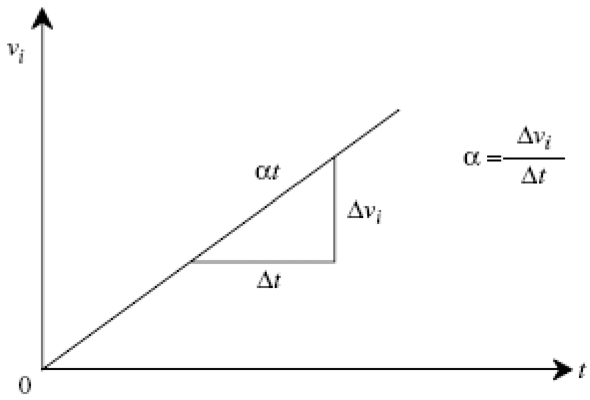


FIGURE 1.18(a) Ramp input

Taking Laplace transforms:

$$\frac{\alpha}{s^2} = \frac{1}{s\tau} v_o(s) + v_o(s) = v_o(s)\left(1 + \frac{1}{s\tau}\right)$$

Multiplying throughout by *s*:

$$\frac{\alpha}{s} = v_o(s)\left(s + \frac{1}{\tau}\right) \qquad (63)$$

Therefore,

$$v_o(s) = \frac{\alpha}{s\left(s + \frac{1}{\tau}\right)} = \frac{A}{s} + \frac{B}{\left(s + \frac{1}{\tau}\right)}$$

From which, $A = \alpha\tau$ and $B = -\alpha\tau$

$$v_o(s) = \frac{\alpha\tau}{s} - \frac{\alpha\tau}{\left(s + \frac{1}{\tau}\right)} = \alpha\tau\left[\frac{1}{s} - \frac{1}{\left(s + \frac{1}{\tau}\right)}\right]$$

Taking Laplace inverse:

$$v_o(t) = \alpha\tau\left(1 - e^{-t/\tau}\right) \qquad (64)$$

If $t/\tau << 1$:

$$e^{-t/\tau} = 1 - \frac{t}{\tau} + \frac{t^2}{2\tau^2}$$

Therefore, $v_o(t) = \alpha\tau \left(1 - 1 + \frac{t}{\tau} - \frac{t^2}{2\tau^2}\right)$

$$v_o(t) = \alpha t\left(1 - \frac{t}{2\tau}\right) \qquad (65)$$

The output falls away from the input, as shown in Fig. 1.18(b). From the waveforms in Fig. 1.18(b), we see that for the output to be the same as the input, $\tau >> T$ (the duration of the ramp). As the value of τ decreases, not only the amplitude of the output decreases but also the signal now is an exponential. The output falls away from the input. So, the choice of τ is dictated by the specific application. Transition error defines deviation from linearity and is given by:

$$e_t = \frac{v_i - v_o}{v_i} \qquad (66)$$

At $t = T$, $v_i = \alpha T$ and $v_o = \alpha T[1 - (T/2\tau)]$. Therefore,

$$e_t = \frac{\alpha T - \alpha T\left(1 - \frac{T}{2\tau}\right)}{\alpha T} = \frac{\frac{\alpha T^2}{2\tau}}{\alpha T} = \frac{T}{2\tau} \qquad (67)$$

Thus,

$$e_t = \frac{T}{2\tau} = \pi f_1 T \quad \text{as} \; \frac{1}{2\tau} = \pi f_1 \qquad (68)$$

The transmission error, e_t describes how faithfully the signal is transmitted to the output. As the input is a ramp and if the output falls away from the input, e_t specifies the deviation from linearity. Let us try to plot the output by considering an example.

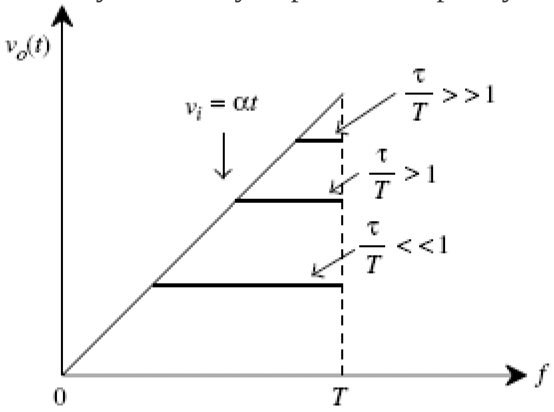


FIGURE 1.18(b) The response of a high-pass circuit to ramp input

EXAMPLE

Example 7: A ramp is applied to an *RC* differentiator, [see Fig.1.1(a)]. Draw to scale the output waveform for the following cases: (i) $T = RC$, (ii) $T = 0.5RC$, (iii) $T = 10RC$.

Solution:

From Eq. (64):

$$v_o = \alpha\,\tau\left(1 - e^{-t/\tau}\right) \qquad v_o = V\left(\frac{\tau}{T}\right)\left(1 - e^{-t/\tau}\right) \quad \text{as} \quad \alpha = \frac{V}{T}$$

The peak of the output will occur at $t = T$.

$$v_o(\text{peak}) = V\left(\frac{\tau}{T}\right)\left(1 - e^{-T/\tau}\right)$$

1. When $T = \tau$, $(\tau/T) = 1$ and $(T/\tau) = 1$

   $$v_o(\text{peak}) = V(1)\left(1 - e^{-1}\right) = 0.632 \text{ V}$$

2. When $T = 0.5\tau$

   $$\left(\frac{T}{\tau}\right) = 0.5 \quad \text{and} \quad \left(\frac{\tau}{T}\right) = 2$$

   $$v_o(\text{peak}) = V(2)\left(1 - e^{-0.5}\right) = 0.788 \text{ V}$$

3. When $T = 10\tau$

   $$\left(\frac{T}{\tau}\right) = 10 \quad \left(\frac{\tau}{T}\right) = 0.1$$

$v_o(peak) = V\,(0.1)\left(1 - e^{-10}\right) = V(0.1)(1 - 0.000045) = 0.1$ V

The response is plotted in Fig. 1.19.

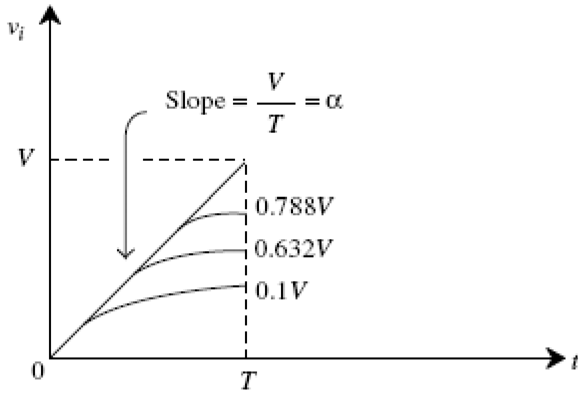


FIGURE 1.19 The response of the high-pass circuit to ramp input

2. DIFFERENTIATORS

Sometimes, a square wave may need to be converted into sharp positive and negative spikes (pulses of short duration). By eliminating the positive spikes, we can generate a train of negative spikes and vice-versa. The pulses so generated may be used to trigger a multivibrator. In such cases, a differentiator is used. If in a circuit, the output is a differential of the input signal, then the circuit is called a differentiator.

A High-pass RC *Circuit as a Differentiator*

If the time constant of the high-pass *RC* circuit, shown in Fig. 1.1(a), is much smaller than the time period of the input signal, then the circuit behaves as a differentiator. If T is to be large when compared to τ, then the frequency must be small. At low frequencies, X_C is very large when compared to R. Therefore, the voltage drop across R is very small when compared to the drop across C.

$$v_i = \frac{1}{C}\int i\,dt + iR$$

But $iR = v_o$ is small. Therefore,

$$v_i = \frac{1}{C}\int i\,dt \quad or \quad v_i = \frac{1}{\tau}\int v_o\,dt \quad (\text{since } i = V_o/R)$$

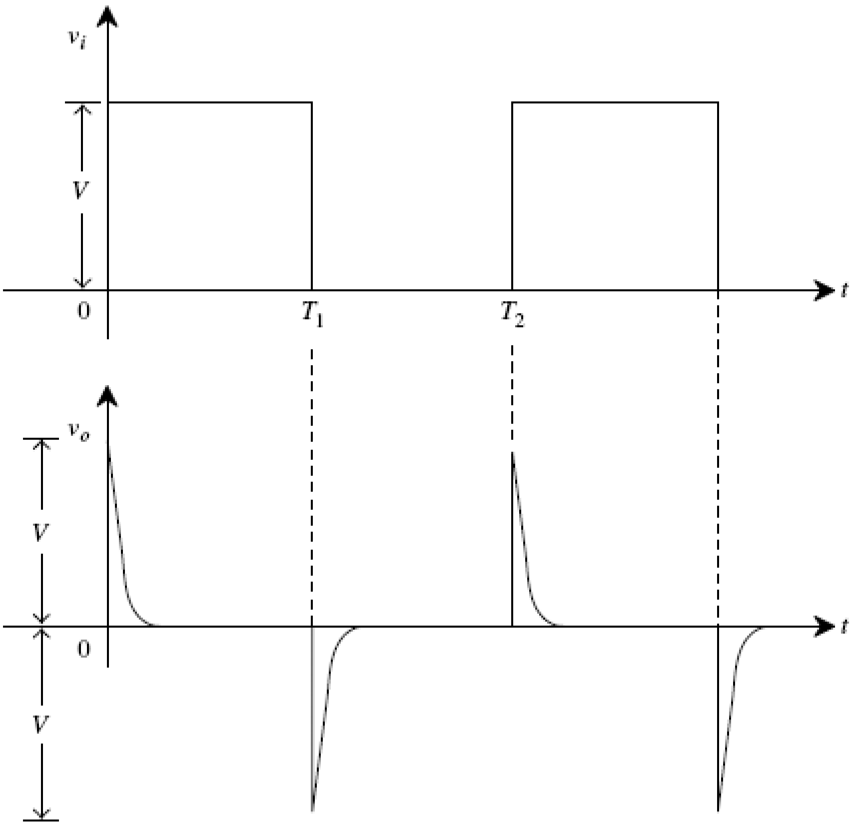


FIGURE 1.20 The output of a differentiator

Differentiating:

$$\frac{dv_i}{dt} = \frac{v_o}{\tau}$$

$$v_o = \tau \frac{dv_i}{dt} \tag{69}$$

Therefore,

$$v_o \propto \frac{dv_i}{dt} \tag{70}$$

Thus, from Eq. (70), it can be seen that the output is proportional to the differential of the input signal, as shown in Fig. 1.20. If the input $v_i(t) = V_m \sin \omega t$:

$$v_o(t) \approx RC\left[\frac{d}{dt}(V_m \sin \omega t)\right]$$

$$v_o(t) \approx V_m \frac{\omega}{\omega_1} \quad \text{where } \omega_1 = \frac{1}{RC}$$

We have from Eq. (3):

$$\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega_1}{\omega}\right)^2}} \quad \text{and} \quad \theta = \tan^{-1}(\omega_1/\omega)$$

When θ = 90°, the sine function at the input becomes a cosine function at the output, as is required in a differentiator. When ω_1/ω = 100, θ = 89.4° which is nearly equal to 90°. Hence, a high-pass circuit behaves as a good differentiator only when $RC << T$, and the output is a co-sinusiodally varying signal if the input is a sine wave. If the input is a square wave, the output is in the form of positive and negative spikes, as shown in Fig.1.20.

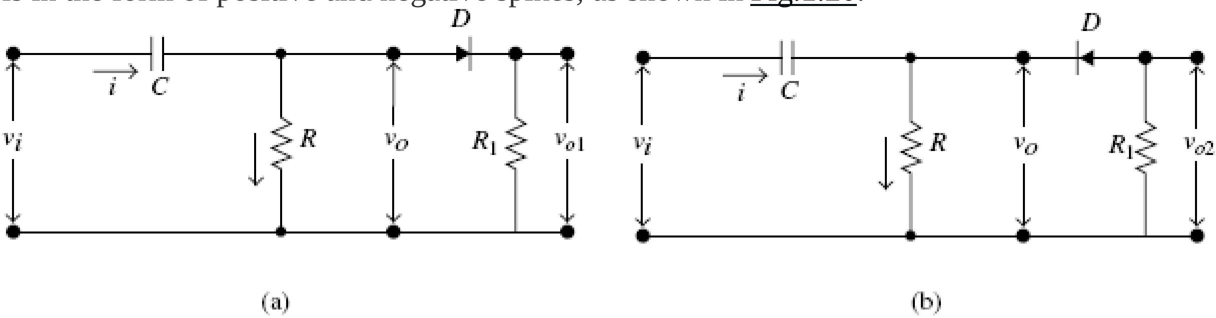


FIGURE 1.21(a) The differentiator circuit when: negative spikes are eliminated in the output; and (b) positive spikes are eliminated in the output

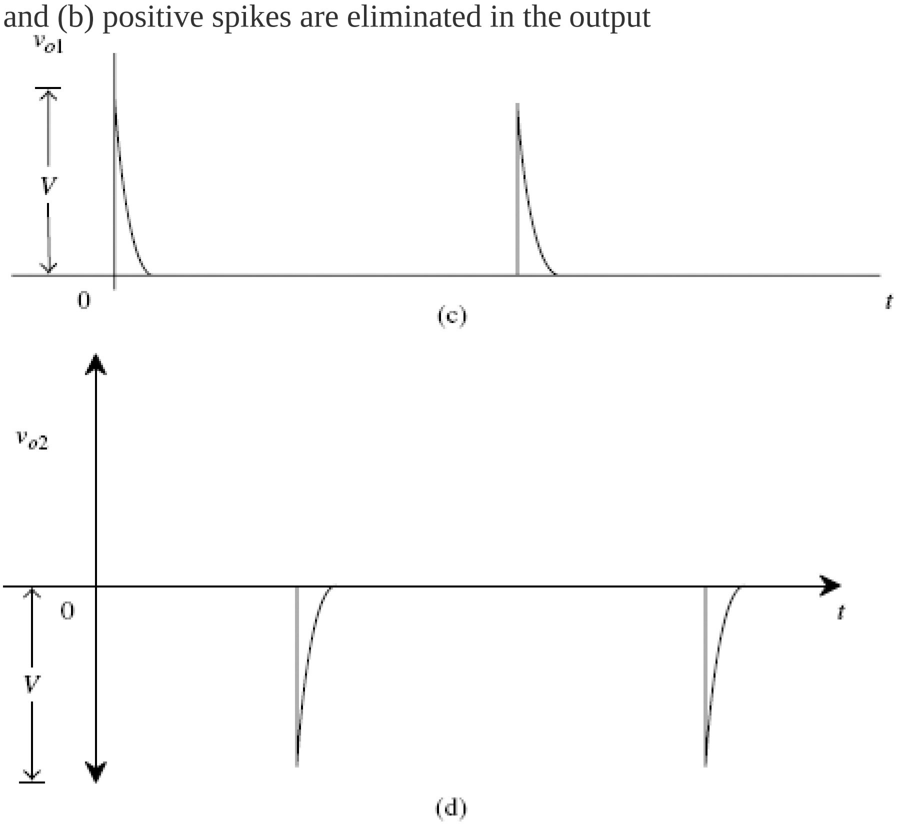


FIGURE 1.21(c) The output of a differentiator showing: positive spikes only; and (d) negative spikes only

The output of the differentiator in Fig. 2.20 contains both positive and negative spikes. If only positive spikes are needed to trigger a multivibrator (to be considered later), we use the circuit shown in Fig. 1.21(a). Here, since D conducts only when the input spikes are positive, the negative spikes are eliminated. Alternately, if only negative spikes are needed, the positive spikes are eliminated using the circuit in Fig. 1.21(b), since *D* conducts only when the input spikes are negative. The output of the circuit in Fig. 1.21(a) is shown in Fig. 1.21(c). Similarly, the output of the circuit in Fig. 1.21(b) is shown in Fig. 1.21(d).

An Op-amp as a Differentiator

An operational amplifier, commonly known as an op-amp, can be used as a differentiator, as shown inFig. 1.22(a).

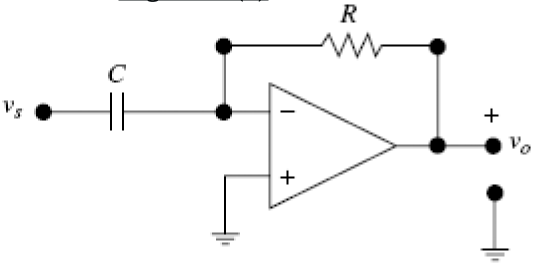


FIGURE 1.22(a) Op-amp as a differentiator

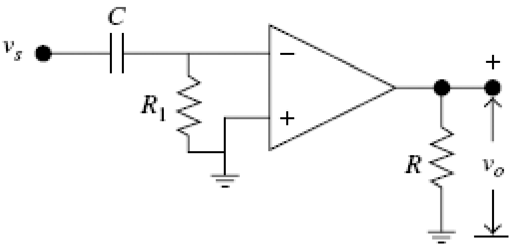


FIGURE 1.22(b) The op-amp differentiator circuit resulting from the use of Miller's theorem

From Miller's theorem:

$$Z_1 = \frac{Z'}{1-A} \quad \text{and} \quad Z_2 = \frac{Z'A}{A-1}$$

where *A* is the gain of the amplifier. The resistance, *R,* appears between the input and output terminals of the op-amp. Using Miller's theorem, *R* can be replaced by R_1 and R_2 as $R_1 = R/(1-A)$ is small since *A*is large; and $R_2 = RA/(A-1) = R$ since *A* is large. Hence, the op-amp circuit can be redrawn as shown in Fig. 1.22(b).

For a good differentiator, τ (= R_1C) should be small. As R_1 is a very small value of resistor (since *A* is large), an op-amp differentiator behaves as a better differentiator when compared to a simple *RC* differentiator, without physically reducing the value of *R*.

Double Differentiators

The circuit in Fig. 1.23 is called a double differentiator as we have two high-pass differentiating circuits. In the figure, *A* is the gain of the inverting amplifier. Here, $R_1C_1 = \tau_1$ and $R_2C_2 = \tau_2$ are small when compared to the time period of the input signal.

Let the input to the circuit be a ramp, i.e., $v_i = \alpha t$. From Eq. (64), the output of the first high-pass R_1C_1circuit for the ramp input is:

Therefore, the output voltage of the amplifier *v* is written as:

$$v = -A\alpha\tau_1\left(1 - e^{-t/\tau_1}\right) \qquad (72)$$

where, *A* is the amplifier gain. It can be seen from Eq. (72) that *v* has phase inversion. The output of the first high-pass circuit, which is an exponential, is the input to the second differentiator. We know from Eq. (55) that the output of this second differentiator is a pulse.

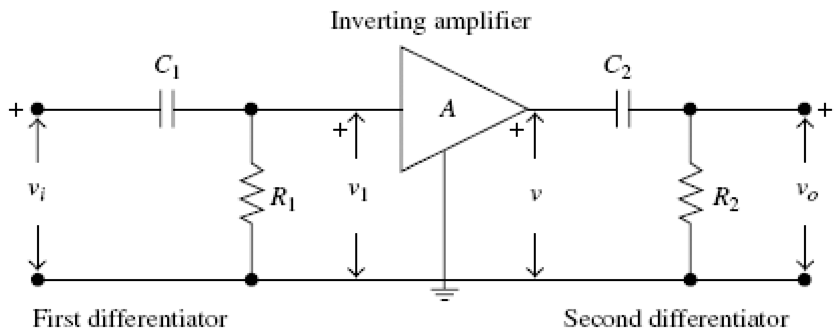


FIGURE 1.23 A double differentiator

$$v_o = -A\alpha\tau_1 \frac{n}{n-1}\left(e^{-x/n} - e^{-x}\right)$$

where $n = \tau_2/\tau_1$ and $x = t/\tau_1$. So,

$$v_o = A\alpha\tau_1 \frac{n}{n-1}\left(e^{-x} - e^{-x/n}\right) \quad (73)$$

Therefore,

$$v_o = v\frac{n}{n-1}\left(e^{-x} - e^{-x/n}\right) \quad (74)$$

For $n = 1$

$$v_o = vxe^{-x} = A\alpha\tau(t/\tau)e^{-t/\tau} = A\alpha te^{-t/\tau} \quad (75)$$

The ramp voltage which is input to the double differentiator is converted to a pulse. The response is plotted in Fig. 1.24. From Eq. (1.75), the output for $\tau = \tau_1 = \tau_2$ is given as:

$$v_o = A\alpha te^{-t/\tau} \quad (76)$$

From Eq. (72) the output of the amplifier v is given as:

$$v = -A\alpha\tau_1\left(1 - e^{-t/\tau_1}\right)$$

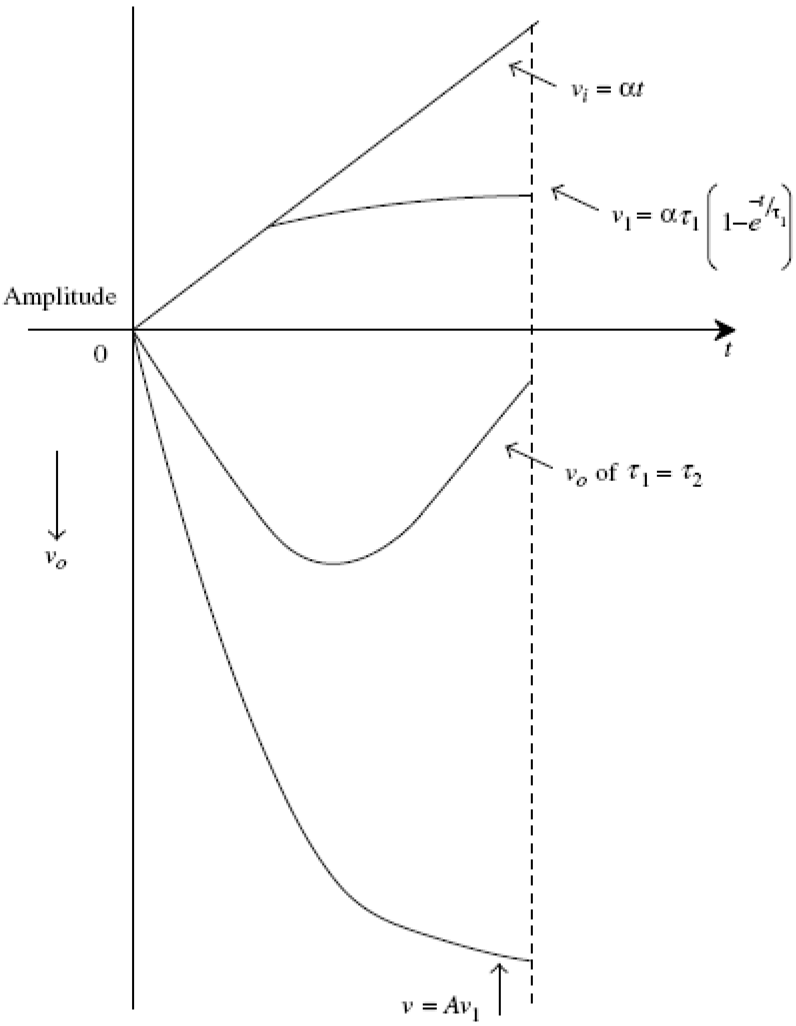


FIGURE 1.24 The response of a double differentiator to a ramp input

The initial slope of this output is:

$$\frac{dv}{dt}\Big|_{t=0} = A\alpha \qquad (77)$$

From Eq. (71), we have:

$v_1 = \alpha\tau\,(1 - e^{-t/\tau})$

The initial slope of the output of the first differentiator (input to the amplifier) is:

$$\frac{dv_1}{dt}\Big|_{t=0} = \alpha \qquad (78)$$

We see from Eqs. (77) and (78), the initial slope of the input to the amplifier v_1 is α, whereas the initial slope of the amplifier output v is $A\alpha$. The output rises much faster than the input, as shown in Fig. 1.24. Hence, the amplifier is called a rate-of-rise amplifier. At this point, it is relevant to talk about a comparator. A circuit that compares the input with a reference and tells us the instant at which the input has reached the reference level is called a comparator. One such simple and practical comparator is a diode comparator. Sometimes a circuit needs to be activated the moment the input reaches a predetermined level. The diode comparator will not be able to do this job. Thus, the output of the diode comparator is given as the input to the double differentiator. As the output of the double differentiator is a pulse whose amplitude and duration can be controlled, this output can activate the desired circuit. We discuss this aspect in greater details in later chapters.

2.1 THE RESPONSE OF A HIGH-PASS *RL* CIRCUIT TO STEP INPUT

A high-pass *RL* circuit is represented in Fig. 2.2(b). If a step of magnitude *V* is applied, let us find the response. Writing the KVL equation:

$$v_i = iR + L\frac{di}{dt} \qquad (79)$$

$$v_o = L\frac{di}{dt} \qquad (80)$$

Therefore, from Eq. (80)

$$\frac{di}{dt} = \frac{v_o}{L} \quad di = \frac{1}{L}v_o dt \quad i = \frac{1}{L}\int v_o dt$$

As $v_i = V$, Eq. (2.79) can also be written as:

$$V = \frac{R}{L}\int v_o dt + v_o$$

Applying Laplace transforms:

$$\frac{V}{s} = \left(\frac{1}{s\tau} + 1\right) v_o(s) \quad \text{where } \tau = \frac{L}{R}$$

$$V = \left(s + \frac{1}{\tau}\right) v_o(s) \qquad v_o(s) = \frac{V}{\left(s + \frac{1}{\tau}\right)}$$

Taking Laplace inverse:

$$v_o(t) = Ve^{-t/\tau} \qquad (81)$$

Similarly, the response of this circuit is evaluated for other inputs. This high-pass circuit is used as a differentiator if $L/R \ll T$. Since $v_o = L di/dt$, and $i \approx v_i/R$:

$$v_o = \frac{L}{R}\frac{dv_i}{dt} \qquad (82)$$

SOLVED PROBLEMS

Example 8: The output of a step generator has an amplitude of 10 V and rise-time of 1.1 ns. When this is applied as an input to a high-pass circuit with $R = 100\ \Omega$ [see Fig 1.25(a)], there appears across *R* a pulse of amplitude 1 V. Find the value of the capacitance.

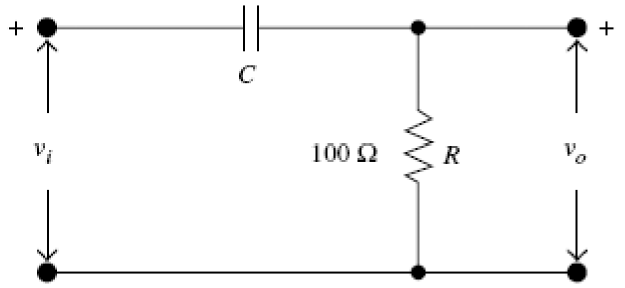


FIGURE 1.25(a) The given coupling circuit

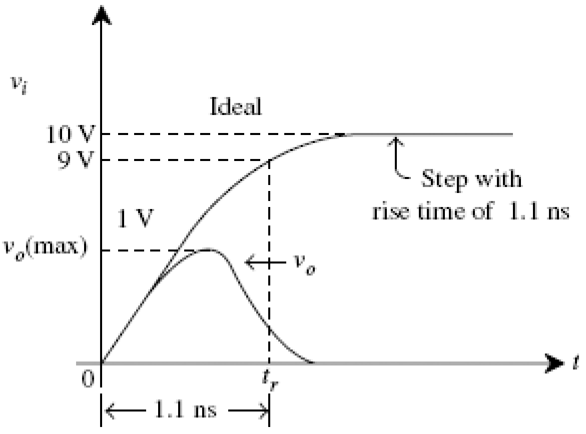


FIGURE 1.25(b) The response of the circuit

Solution:

The response of the circuit in Fig. 1.25(a) is shown in Fig. 1.25(b).
Rise time of step input $t_r = 2.2\ \tau_1 = 1.1$ ns
Therefore, the time constant of the exponential input:

$$\tau_1 = \frac{1.1}{2.2} = 0.5\,\text{ns}$$

The maximum value of the output is:

$v_o(\max) = v_i\, n^{1/(1-n)} = 1\text{V}$
$10n^{1/(1-n)} = 1$ or
$n = 0.14$

$$n = \frac{\tau}{\tau_1} = 0.14$$

$\tau = n\tau_1 = 0.14 \times 0.5$ ns $= 0.07$ ns

$\tau = RC = 0.07 \times 10^{-9}$ s

Therefore,

$$C = \frac{0.07 \times 10^{-9}}{100} = 0.7\,\text{pF}$$

Example 9: A limited ramp from a generator rises linearly to V_s in a time period $T_s = 0.1\ \mu$s and remains constant for 2 μs. This signal is applied to a differentiating circuit whose time constant is 0.01μs. The resultant pulse at the output of the differentiator has a maximum value of 15 V. What is the peak amplitude of the ramp at the output of the generator?
Solution:

$RC = \tau = 0.01\ \mu s = 0.01 \times 10^{-6}$ s

$v_o(\max) = 15$ V

$$v_o(t) = \alpha\tau\left(1 - e^{-T_s/\tau}\right)$$

$\alpha\tau = v_o(\max) = 15$ V

$$\alpha = \frac{15}{\tau} = \frac{15}{0.01 \times 10^{-6}}\ \text{V/s}$$

$T_s = 0.1\ \mu$s

The peak value of the ramp from the generator is:

$$V_s = \alpha T_s = \frac{15}{0.01 \times 10^{-6}} \times 0.1 \times 10^{-6} = 150\ \text{V}$$

The input and output are plotted in Fig. 2.26.

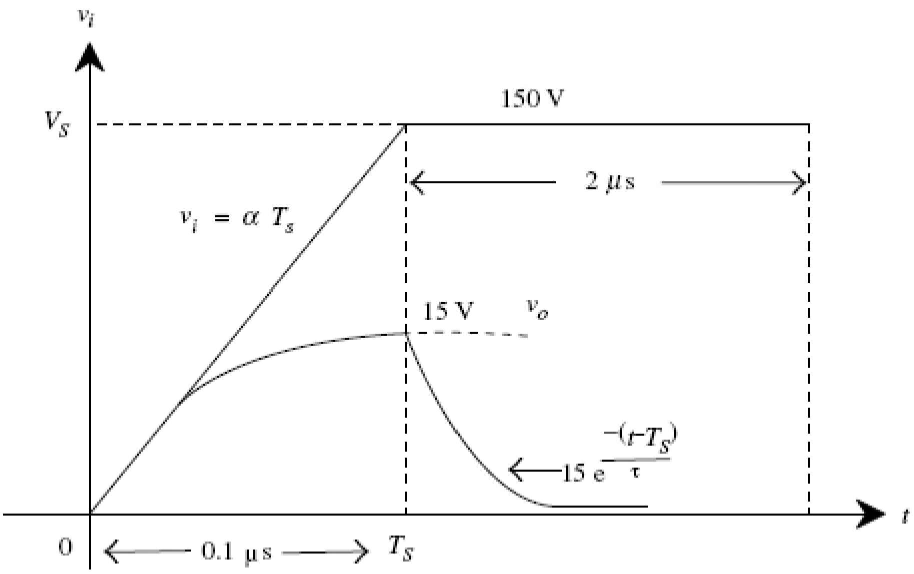


FIGURE 1.26 The input and the output for the specified conditions

3) LOWPASS CIRCUIT

3.1 INTRUDUCTION

A low-pass circuit is one which gives an appreciable output for low frequencies and zero or negligible output for high frequencies. In this chapter, we essentially consider low-pass *RC* and *RL* circuits and their responses to different types of inputs. Also, we study attenuators that reduce the magnitude of the signal to the desired level. Attenuators which give an output that is independent of frequency are studied. One application of such a circuit is as a CRO probe. Further, the response of the *RLC* circuit to step input is considered and its output under various conditions such as under-damped, critically damped and over-damped conditions is presented. The application of an *RLC* circuit as a ringing circuit is also considered.

3.2 LOW-PASS CIRCUITS

Low-pass circuits derive their name from the fact that the output of these circuits is larger for lower frequencies and vice-versa. Figures 3.1(a) and (b) represent a low-pass *RC* circuit and a low-pass *RL* circuit, respectively.

In the *RC* circuit, shown in Fig. 3.1(a), at low frequencies, the reactance of *C* is large and decreases with increasing frequency. Hence, the output is smaller for higher frequencies and vice-versa. Similarly, in the *RL* circuit shown in Fig. 3.1(b), the inductive reactance is small for low frequencies and hence, the output is large at low frequencies. As the frequency increases, the inductive reactance increases; hence, the output decreases. Therefore, these circuits are called low-pass circuits. Let us consider the response of these low-pass circuits to different types of inputs.

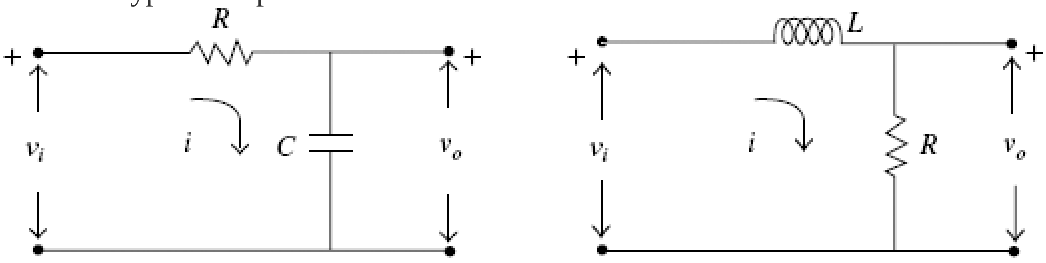


FIGURE 3.1(a) A low-pass *RC* circuit; and (b) a low-pass *RL* circuit

3.2.1 The Response of a Low-pass *RC* Circuit to Sinusoidal Input

For the circuit given in Fig. 3.1(a), if a sinusoidal signal is applied as the input, the output v_o is given by the relation:

$$v_0 = v_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \qquad \frac{v_o}{v_i} = \frac{1}{1 + j\omega CR}$$

$$\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{1 + (\omega CR)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_2}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\tau}{T}\right)^2}} \qquad \textbf{(3.1)}$$

where, $\omega_2 = 1/CR = 1/\tau$. From Eq. (3.1), the phase shift θ the signal undergoes is given as:

$\theta = \tan^{-1}(\omega/\omega_2) = \tan^{-1}(\tau/T)$

Figure 3.2(a) shows a typical frequency vs. gain characteristic. Hence, f_2 is the upper half-power frequency. At $\omega = \omega_2$,

$$\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{2}} = 0.707$$

Figure 3.2(b) shows the variation of gain with frequency for different values of τ. As is evident from the figure, the half-power frequency, f_2, increases with the decreasing values of τ, the time constant. The sinusoidal signal undergoes a change only in the amplitude but its shape remains preserved.

Figure 3.2(c) shows the variation of θ as a function of frequency. As (τ/T) becomes large, θ approaches 90°. This characteristic can be appreciated when we talk about an integrator later.

3.2.2 The Response of a Low-pass *RC* Circuit to Step Input

Let a step voltage be applied as the input to the low-pass *RC* circuit shown in Fig. 3.1(a). The output v_o can be obtained by using Eq. (2.9) as shown in Fig. 3.3. We have $RC = \tau$.

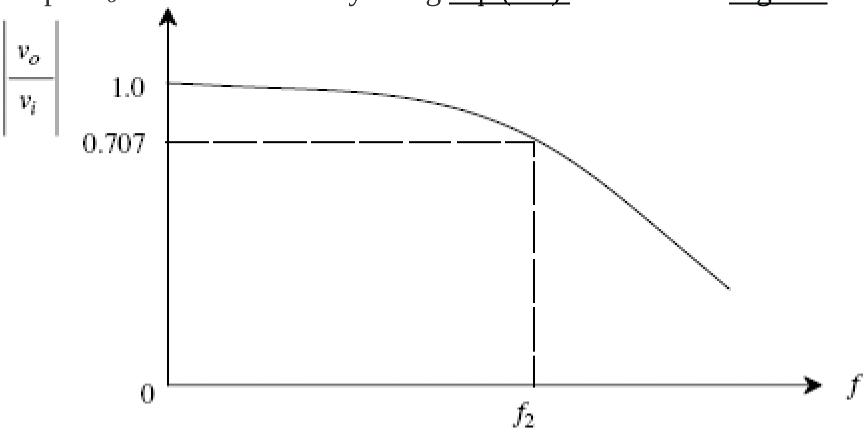


FIGURE 3.2(a) Typical frequency-vs-gain characteristic of a low-pass circuit to sinusoidal input

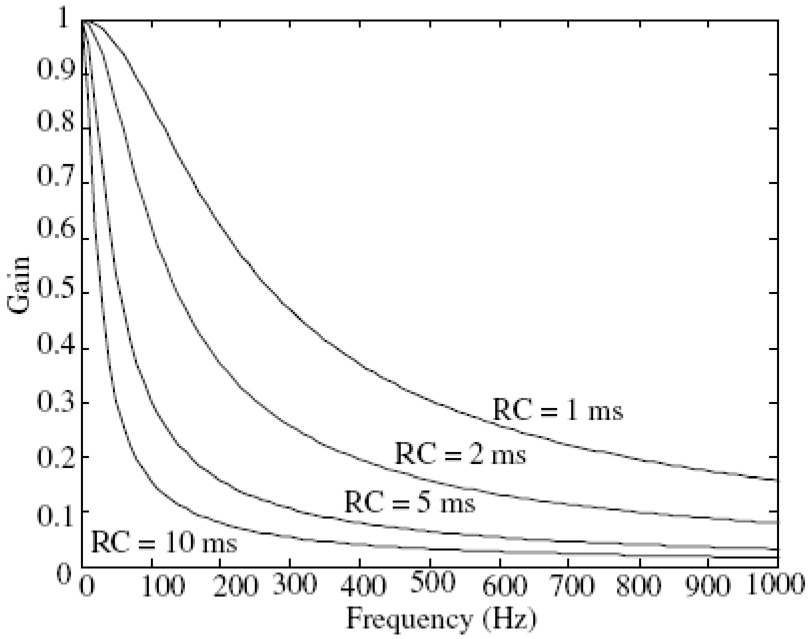


FIGURE 3.2(b) Gain-vs-frequency curves for different values of τ

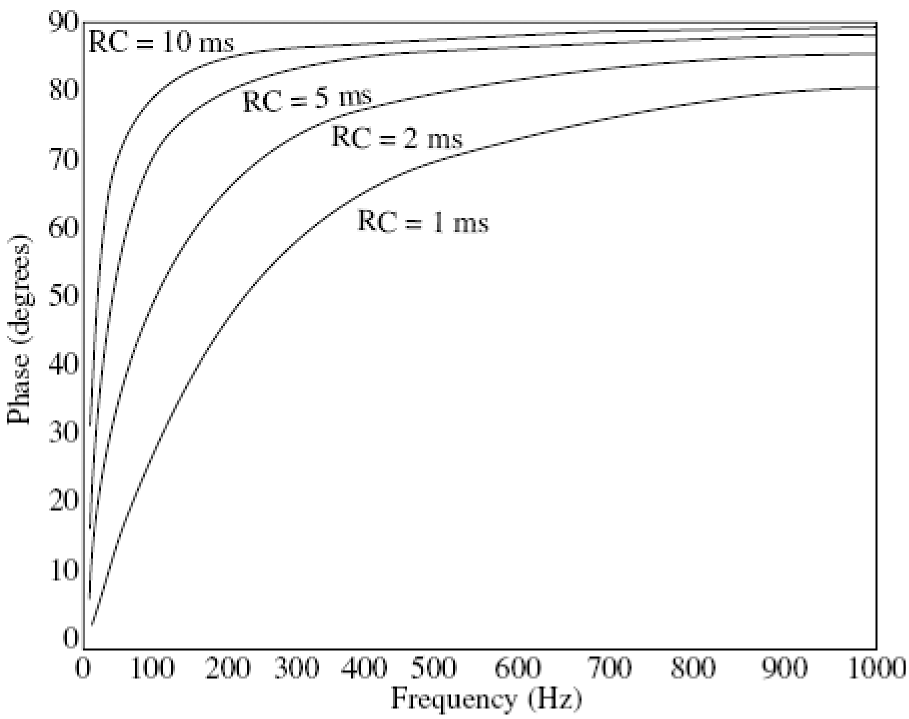


FIGURE 3.2(c) Phase-vs-frequency curves for different values of τ

$v_o(t) = v_f + (v_i - v_f)\, e^{-t/\tau}$

Here, $v_f = V$ and $v_i = 0$. Therefore,

$$v_o(t) = V - Ve^{-t/\tau} = V\left(1 - e^{-t/\tau}\right) \tag{3.2}$$

As $t \to \infty$, $v_o(t) \to V$.

Initially, as the capacitor behaves as a short circuit, the output voltage is zero. As the capacitor charges, the output reaches the steady-state value of V in a time interval that is dependent on the time constant, τ. On the other hand, the output of Eq. (3.2) can also be obtained by solving the following differential equation. From Fig. 3.1(a), For $v_i = V$:

$$V = v_i = Ri + \frac{1}{C}\int i\, dt \tag{3.3}$$

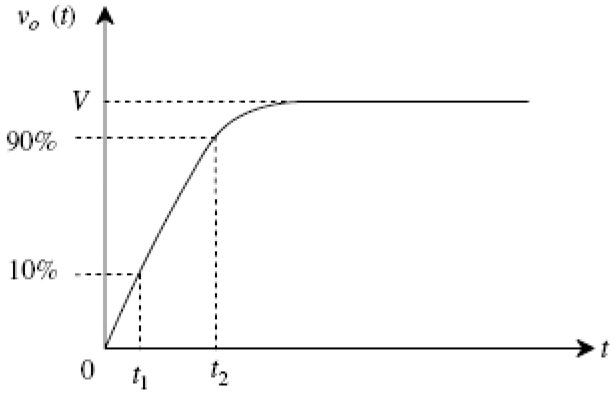


FIGURE 3.3 The response of a low-pass circuit to step input

We know that $(1/C)\int i dt = v_o$

$$\frac{i}{C} = \frac{dv_o}{dt} \quad \text{or} \quad i = C\frac{dv_o}{dt} \tag{3.4}$$

From Eqs. (3.3) and (3.4):

$$V = RC\frac{dv_o}{dt} + v_o \qquad V = \tau\frac{dv_o}{dt} + v_o \tag{3.5}$$

Taking Laplace transforms:

$$\frac{V}{s} = \tau s v_o(s) + v_o(s) = v_o(s)[\tau s + 1] = v_o(s)\tau\left(s + \frac{1}{\tau}\right), \quad \text{Hence} \quad v_o(s) = \frac{\frac{V}{\tau}}{s\left(s + \frac{1}{\tau}\right)}$$

Resolving into partial fractions:

$$\frac{\frac{V}{\tau}}{s\left(s + \frac{1}{\tau}\right)} = \frac{A}{s} + \frac{B}{\left(s + \frac{1}{\tau}\right)} \qquad \frac{V}{\tau} = A\left(s + \frac{1}{\tau}\right) + Bs$$

Putting $s = 0$:

$$\frac{V}{\tau} = \frac{A}{\tau} \quad \text{or} \quad A = V$$

Putting $s = -1/\tau$

$$\frac{V}{\tau} = B\left(-\frac{1}{\tau}\right) \quad \text{or} \quad B = -V$$

Therefore,

$$v_o(s) = \frac{V}{s} - \frac{V}{\left(s + \frac{1}{\tau}\right)}$$

Taking the Laplace inverse:

$$v_o(t) = V - Ve^{-t/\tau} = V\left(1 - e^{-t/\tau}\right) \qquad \textbf{(3.6)}$$

Now, for the circuit in Fig. 3.1(b):

$$v_i = L\frac{di}{dt} + iR \qquad \textbf{(3.7)}$$

$$v_o = iR \qquad i = \frac{v_o}{R} \qquad \frac{di}{dt} = \frac{1}{R}\frac{dv_o}{dt} \qquad v_i = \frac{L}{R}\frac{dv_o}{dt} + v_o \qquad V = \tau\frac{dv_o}{dt} + v_o$$

Applying the Laplace transform:

$$\frac{V}{s} = \tau s v_o(s) + v_o(s) = v_o(s)(1 + s\tau) = v_o(s)\tau\left(s + \frac{1}{\tau}\right) \qquad v_o(s) = \frac{\frac{V}{\tau}}{s\left(s + \frac{1}{\tau}\right)}$$

$$v_o(t) = V(1 - e^{-t/\tau}) \qquad \textbf{(3.8)}$$

From Eq. (3.8), it may be seen that the output reaches the steady-state value faster for smaller values of τ. Similarly, when τ is large, it takes a longer time for the output to reach the steady-state value.

Rise time: The time taken for the output to reach 90 per cent of its final value from 10 per cent of its final value is called the rise time. Using Eq. (3.8) to calculate the rise time for this circuit:

$$\frac{v_o}{V} = 1 - e^{-t/\tau}$$

From Fig. 3.3 at $t = t_1$, $v_o = 0.1\ V$. Therefore,

$$0.1 = 1 - e^{-t_1/\tau}$$

$$e^{-t_1/\tau} = 0.9$$

$$t_1 = 0.1\tau \qquad (3.9)$$

Similarly at $t = t_2$, $v_o = 0.9\ V$:

$$0.9 = 1 - e^{-t_2/\tau} \qquad e^{-t_2/\tau} = 0.1$$

$$t_2 = 2.3\tau \qquad (3.10)$$

Using Eqs. (3.9) and (3.10), rise time is given as:

$$t_r = t_2 - t_1 = 2.3\tau - 0.1\tau = 2.2\tau \qquad (3.11)$$

Also $f_2 = 1/2\pi RC$

$$RC = \tau = \frac{1}{2\pi f_2}$$

Therefore,

$$t_r = 2.2\tau = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} \qquad (3.12)$$

Let a step voltage V_i be applied to a low-pass circuit. The output does not reach the steady-state value *Vi* instantaneously as desired. Rather, it takes a finite time delay for the output to reach V_i, depending on the value of the time constant of the low-pass circuit employed. If this output is to drive a transistor from the OFF to the ON state, this change of state does not occur immediately, because the output of the low-pass circuit takes some time to reach *Vi*. The transistor is thus said to be switched from the OFF state into the ON state only when the voltage at the output of the low-pass circuit is 90 per cent of *Vi*. If this time delay is to be small, τ should be small. On the contrary, if the output is to be ramp, τ should be large.

3.2.3 The Response of a Low-pass *RC* Circuit to Pulse Input

Let the input to the low-pass circuit be a positive pulse of duration t_p and amplitude V as shown in Fig. 3.5(a). If this positive pulse is applied to drive an *n–p–n* transistor from the OFF state into the ON state, the transistor will be switched ON only after a time delay. Similarly, at the end of the pulse, the transistor will not be switched immediately into the OFF state, but will take a finite time delay. To know how quickly it is possible to switch a transistor from one state to the other, we have to consider the response of a low-pass circuit to the pulse input. During the period 0 to t_p, the input is a step and the output is given by Eq. (3.8). At $t = t_p$ the input falls and the output decays exponentially as given in Eq. (3.13).

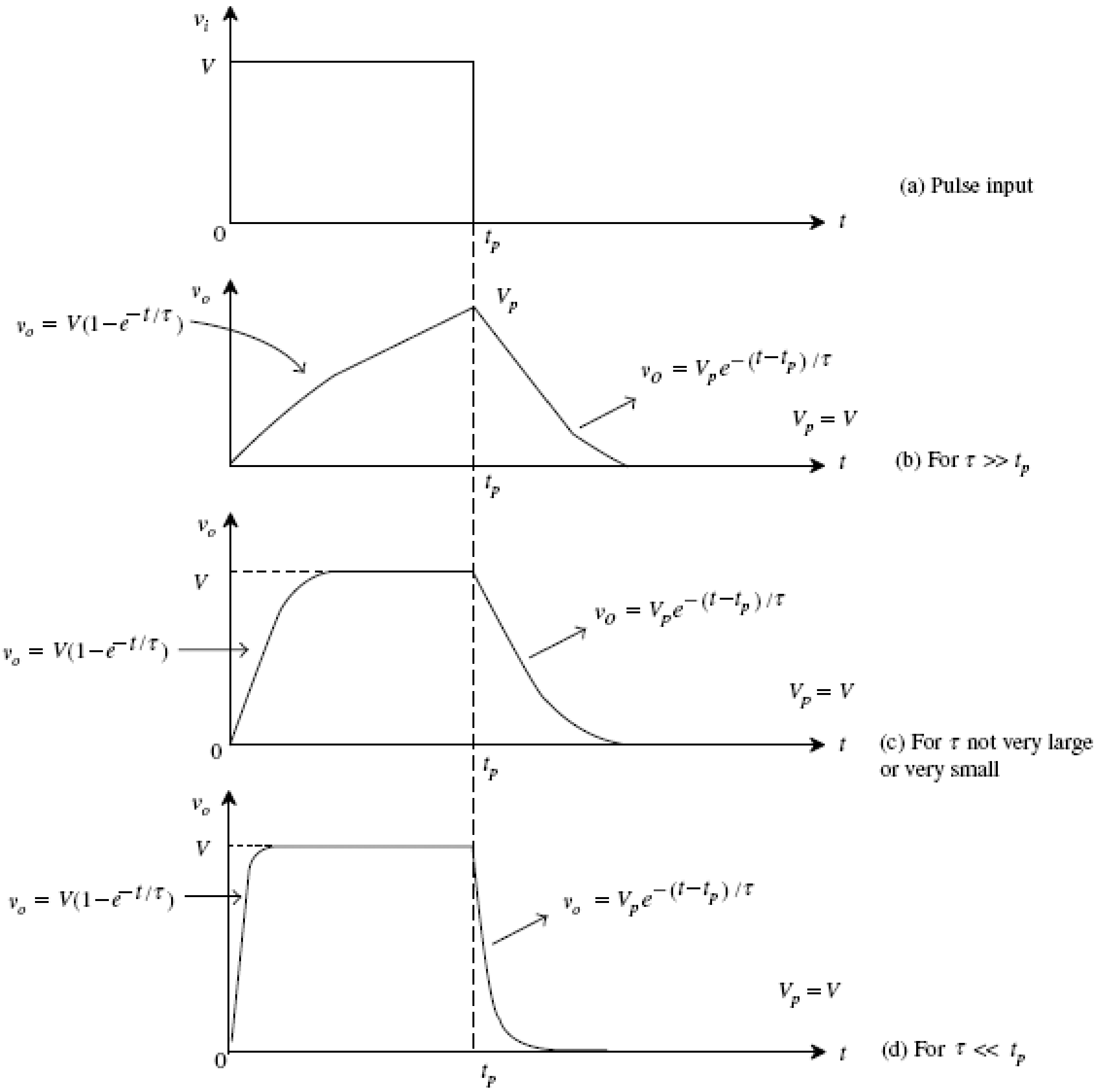


FIGURE 3.5 Response of a low-pass circuit for the pulse input for varying τ

$$v_o(t > t_p) = V_p\, e^{-(t-t_p/\tau)} \qquad \textbf{(3.13)}$$

For $v_i = V$, the output for different values of τ is plotted in Fig. 3.5. It is seen here that the shape of the pulse at the output is preserved if the time constant of the circuit is much smaller than t_p, i.e., $\tau \ll t_p$. However, if a ramp is to be generated during the period of the pulse, τ is chosen such that $\tau \gg t_p$. The method to compute the output is illustrated in Example 3.2.

EXAMPLE

Example 3.2: An ideal pulse of amplitude 10 V is fed to an *RC* low-pass integrator circuit. The width of the pulse is 3 μs. Draw the output waveforms for the following upper 3-dB frequencies: (a) 30 MHz, (b) 3 MHz and (c) 0.3 MHz.

Solution: Consider the low-pass circuit in Fig. 3.1(a).

1. At f_2 = 30 MHz

   We know that $f_2 = 1/2\Pi rc$

$$\tau = RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 30 \times 10^6} = 5.3 \text{ ns}$$

$t_r = 2.2\tau = 2.2 \times 5.3 \times 10^{-9} = 11.67$ ns

At $t = t_p$,

$V_p = V(1 - e^{-t_p/\tau}) = 10(1 - e^{-3\times10-6/5.3\times10-9}) = 10$ V

The output is plotted in Fig. 3.6(a).

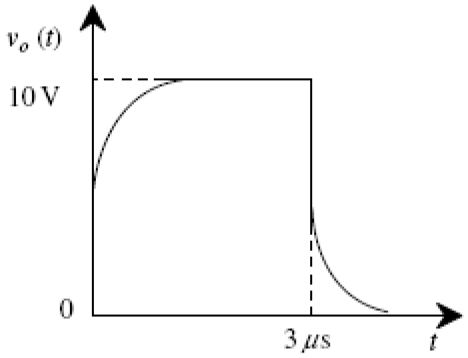


FIGURE 3.6(a) Output waveform at f_2 = 30 MHz

2. At f_2 = 3 MHz

$$\tau = RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 3 \times 10^6} = 53 \text{ ns}$$

$t_r = 2.2\tau = 2.2 \times 53 \times 10^{-9} = 116.6$ ns

At $t = t_p$,

$V_p = V(1 - e^{-t_p/\tau}) = 10(1 - e^{-3\times10-6/53\times10-9}) = 10$ V

The output is plotted in Fig. 3.6(b).

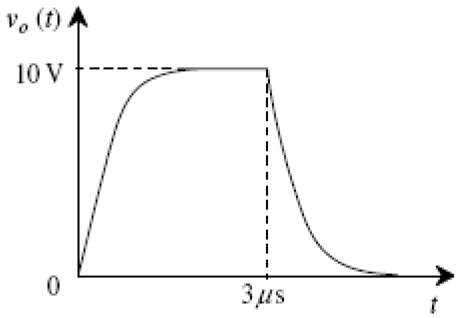


FIGURE 3.6(b) Output waveform at f_2 = 3 MHz

3. At f_2 = 0.3 MHz

$$\tau = RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 0.3 \times 10^6} = 530 \text{ ns}$$

$t_r = 2.2\tau = 2.2 \times 530 \times 10^{-9} = 1.166\ \mu$s

At $t = t_p$,

$_p = V(1 - e^{-t_p/\tau})$

Therefore,

$V_p = 10(1 - e^{-3\times10-6/530\times10-9}) = 9.96$ V

The output is plotted in Fig. 3.6(c).

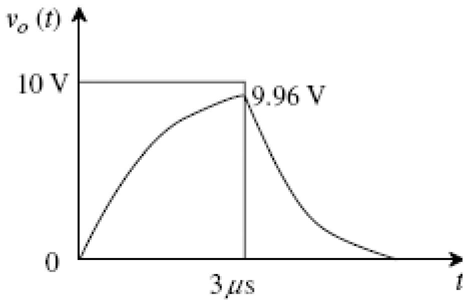


FIGURE 3.6(c) The output waveform at f_2= 0.3 MHz

3.2.4 The Response of a Low-pass *RC* Circuit to a Square-wave Input

Let the input to the low-pass circuit be a square wave as shown in Fig. 3.7 (a).

We have from Eq. (2.9):

$$v_{o1}(t) = v_f + (v_i - v_f)e^{-t/\tau}$$

From Fig. 3.7(c), at $t = T_1$, $v_{o1} = V_2$ and $v_i = V_1$ and $v_f = V'$. Therefore:

$$v_{o1}(T_1) = V_2 = V' + (V_1 - V')e^{-T_1/\tau} \qquad \textbf{(3.14)}$$

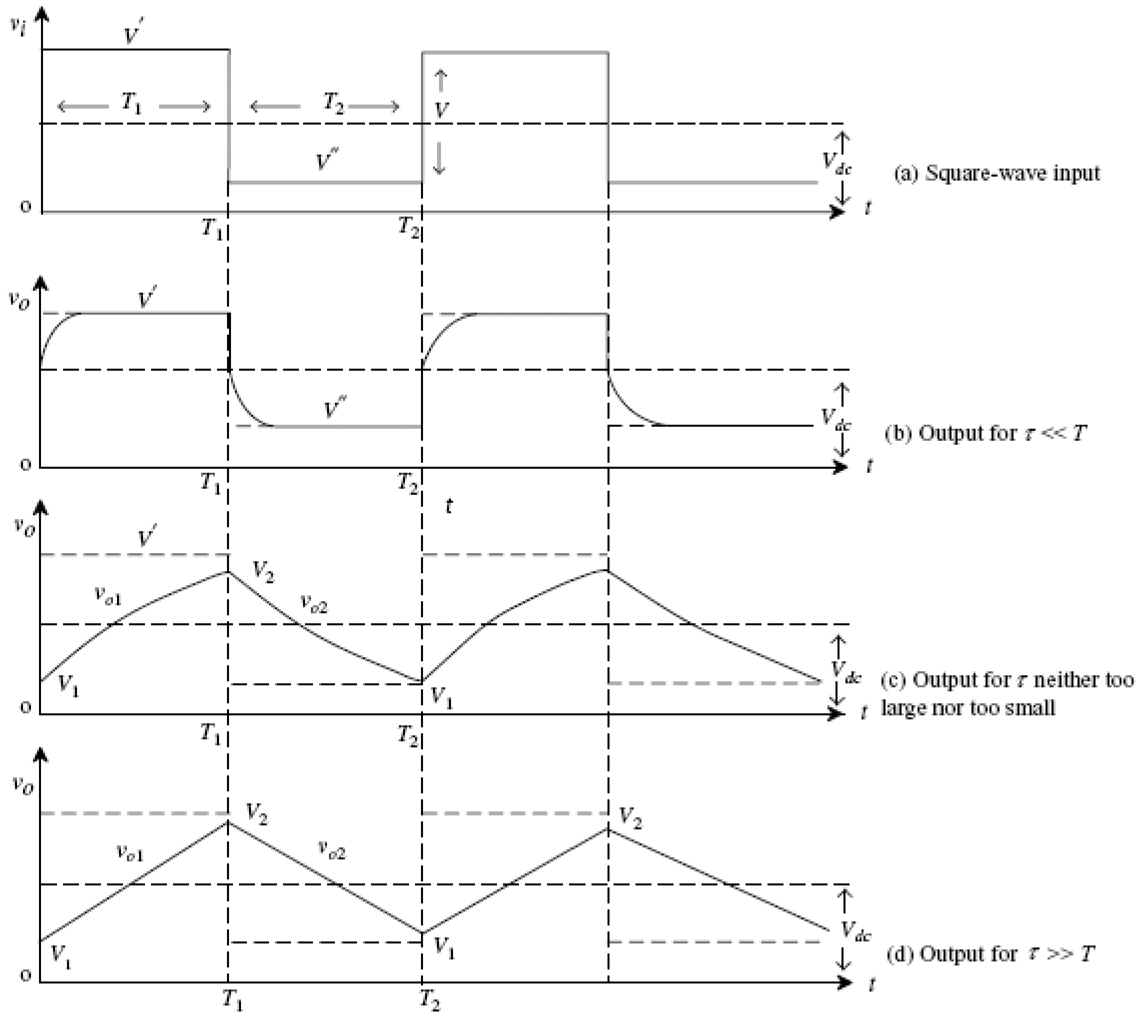


FIGURE 3.7 The response of the low-pass circuit to a square-wave input for different values of τ

Again, at $t = T_2$, $V_{o2} = V_1$ and we have $v_i = V_2$, $V_f = V''$

$$v_{o2}(T_2) = V_1 = V'' + (V_2 - V'')e^{-T_2/\tau} \qquad \textbf{(3.15)}$$

If the input is a symmetric square wave:

$$T_1 = T_2 = \frac{T}{2} \qquad \textbf{(3.16)}$$

Also

$$V' = -V'' = \frac{V}{2} \quad \text{and} \quad V_2 = -V_1 \qquad \textbf{(3.17)}$$

Using Eqs. (3.14) and (3.17):

$$V_2 = \frac{V}{2} + \left(-V_2 - \frac{V}{2}\right)e^{-T/2\tau} \qquad V_2\left(1 + e^{-T/2\tau}\right) = \frac{V}{2}\left(1 - e^{-T/2\tau}\right)$$

$$V_2 = \frac{V}{2}\frac{\left(1 - e^{-T/2\tau}\right)}{\left(1 + e^{-T/2\tau}\right)} \qquad \textbf{(3.18)}$$

$$\therefore V_2 = \frac{V}{2}\ \tanh\ \frac{T}{4\tau} \qquad \textbf{(3.19)}$$

Using Eqs. (3.17) and (3.19), it is possible to calculate V_2 and V_1 and plot the output waveforms as given in Figs. 3.7(c) and (d), respectively.

If $\tau \ll T$, then the wave shape is maintained. And if $\tau \gg T$, the wave shape is highly distorted, but the output of the low-pass circuit is now a triangular wave. So it is possible to derive a triangular wave from a square wave by choosing τ to be very large when compared to $T/2$ of the symmetric square wave.

3.2.9 Low-pass RL Circuits

Consider the low-pass *RL* circuit in Fig. 3.1(b). For this low-pass circuit, $v_o = R/L \int v_i \, dt$. This circuit can also be used as an integrator when the time constant $L/R \gg T$. The major limitations of high-pass and low-pass *RL* circuits are:

1. For a large value of inductance, an iron-cored inductor has to be used. As such it is bulky and occupies more space.
2. Inductors are more lossy elements, when compared to capacitors. So, it is possible to get ideal capacitors, but not ideal inductors.

ATTENUATORS

An attenuator is a circuit that reduces the amplitude of the signal by a finite amount. A simple resistance attenuator is represented in Fig. 3.16. The output of the attenuator shown in Fig. 3.16 is given by the relation:

$$v_o = v_i \times \frac{R_2}{R_1 + R_2} = \alpha \, v_i$$

From this equation, it is evident that the output is smaller than the input, which is the main purpose of an attenuator—to reduce the amplitude of the signal. Attenuators are used when the signal amplitude is very large. Let us measure a voltage, say, 5000 V, using a CRO; such a large voltage may not be handled by the amplifier in a CRO. Therefore, to be able to measure so that the voltage that is actually connected to the CRO is only 500 V. The output of the attenuator is thus reduced depending on the choice of R_1 and R_2.

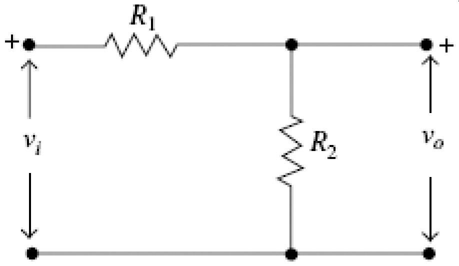


FIGURE 3.16 A resistance attenuator

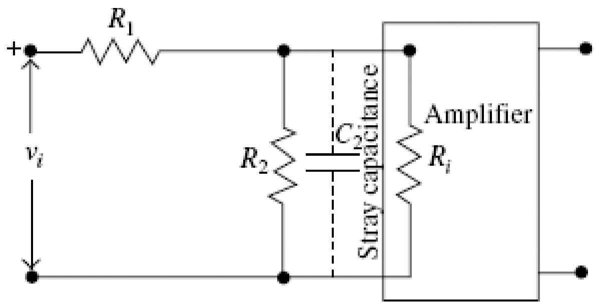


FIGURE 3.17(a) The attenuator output connected to amplifier input

Uncompensated Attenuators

If the output of an attenuator is connected as input to an amplifier with a stray capacitance C_2 and input resistance R_i, as shown in Fig. 3.17(a).

Consider the parallel combination of R_2 and R_i. If the amplifier input is not to load the attenuator output, then R_i should always be significantly greater than R_2. The attenuator circuit is now shown in Fig. 3.17(b).

Reducing the two-loop network into a single-loop network by Thevenizing:

$$V_{Th} = v_i \times \frac{R_2}{R_1 + R_2} = \alpha v_i \quad \text{where} \quad \alpha = \frac{R_2}{R_1 + R_2}$$

And $R_{th} = R_1 \| R_2$

Hence, the circuit in Fig. 3.17(b) reduces to that shown in Fig. 3.17(c).

When the input αv_i is applied to this low-pass RC circuit, the output will not reach the steady-state value instantaneously. If, for the above circuit, $R_1 = R_2 = 1$ MΩ and $C_2 = 20$ nF, the rise time is:

$t_r = 2.2\, R_{th}C_2 = 2.2 \times 0.5 \times 10^6 \times 20 \times 10^{-9}$

$t_r = 22$ms

This means that after a time interval of approximately 22ms after the application of the input αv_i to the circuit, the output reaches the steady-state value. This is an abnormally long delay. An attenuator of this type is called an uncompensated attenuator, i.e., its output is dependent on frequency.

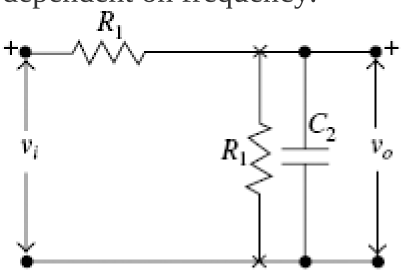


FIGURE 3.17(b) The attenuator, considering the stray capacitance at the amplifier input

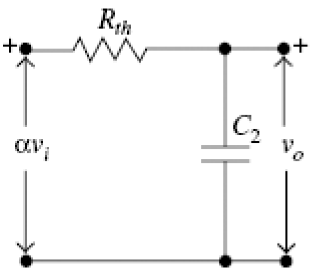


FIGURE 3.17(c) An uncompensated attenuator

Compensated Attenuators

To make the response of the attenuator independent of frequency, the capacitor C_1 is connected across R_1. This attenuator now is called a compensated attenuator shown in Fig. 3.18(a). This circuit in Fig. 3.18(a) is redrawn as shown in Fig. 3.18(b).

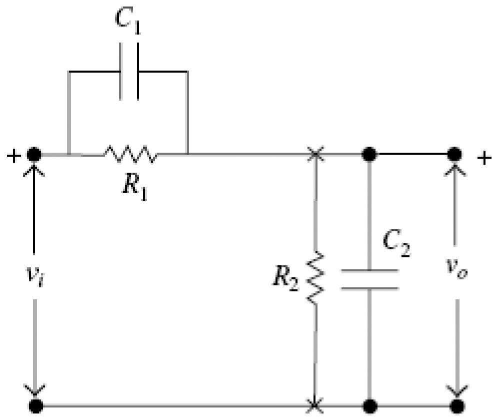


FIGURE 3.18(a) A compensated attenuator

FIGURE 3.18(b) Redrawn circuit of Fig 3.18(a)

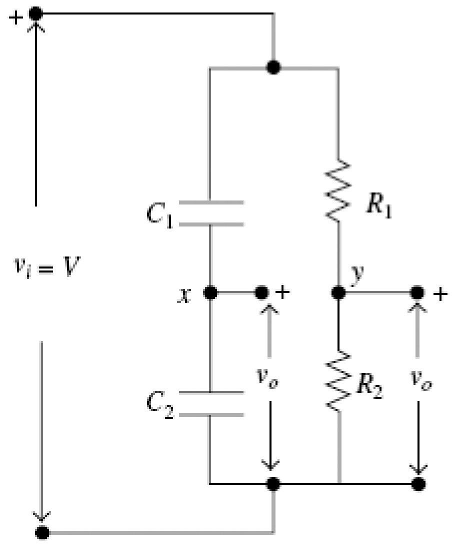


FIGURE 3.18(c) The compensated attenuator open-circuiting the *xy* branch

In Figs. 3.18(a) and (b), R_1, R_2, C_1, C_2 form the four arms of the bridge. The bridge is said to be balanced when $R_1C_1 = R_2C_2$, in which case no current flows in the branch *xy*. Hence, for the purpose of computing the output, the branch *xy* is omitted. The resultant circuit is shown in Fig. 3.18(c).

When a step voltage with $v_i = V$ is applied as an input, the output is calculated as follows: At $t = 0+$, the capacitors do not allow any sudden changes in the voltage; as the input changes, the output should also change abruptly, depending on the values of C_1 and C_2.

$$v_o(0^+) = V\frac{C_1}{C_1 + C_2} \qquad \textbf{(3.43)}$$

Thus, the initial output voltage is determined by C_1 and C_2. As $t \to \infty$, the capacitors are fully charged and they behave as open circuits for dc. Hence, the resultant output is:

$$v_o(\infty) = V\frac{R_2}{R_1 + R_2} \qquad \textbf{(3.44)}$$

Perfect compensation is obtained if, $v_o(0^+) = v_o(\infty)$
From this using Eqs. 3.43 and 3.44 we get:

$$\frac{C_1}{C_1 + C_2}V = V\frac{R_2}{R_1 + R_2} \qquad \textbf{(3.45)}$$

and the output is αv_i.

$$C_1R_1 = C_2R_2 \quad \text{or}$$

$$C_1 = (R_2/R_1)C_2 = C_p \qquad \textbf{(3.46)}$$

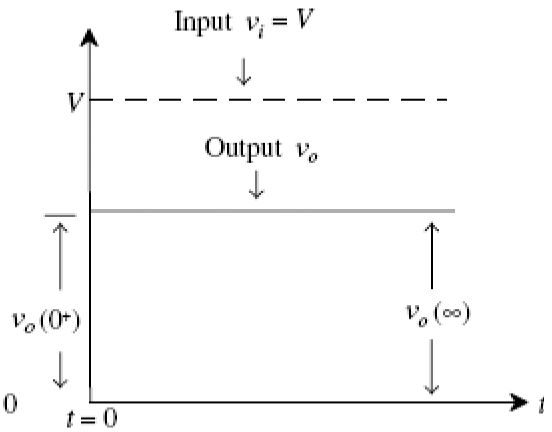


FIGURE 3.19(a) A perfectly compensated attenuator ($C_1 = C_2$)

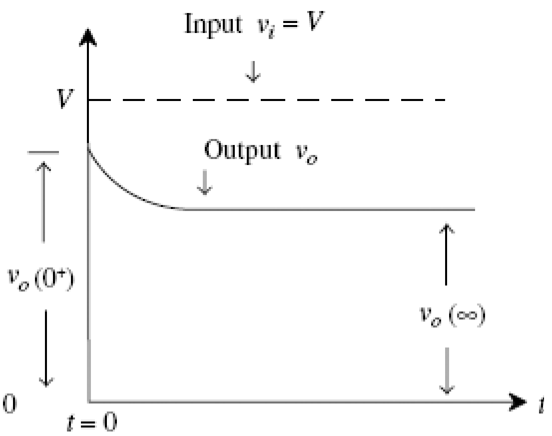


FIGURE 3.19(b) An over-compensated attenuator ($C_1 > C_2$)

Let us consider the following circuit conditions:

1. When $C_1 = C_p$, the attenuator is a perfectly compensated attenuator.
2. When $C_1 > C_p$, it is an over-compensated attenuator.
3. When $C_1 < C_p$, it is an under-compensated attenuator.

The response of the attenuator to a step input under these three conditions is shown in Figs. 3.19(a),(b) and (c), respectively.

In the attenuator circuit, as at t = 0+, the capacitors C_1 and C_2 behave as short circuits, the current must be infinity. But impulse response is impossible as the generator, in practice, has a finite source resistance, not ideally zero. Now consider the compensated attenuator with source resistance R_s [see Fig. 3.19(d)].

If the xy loop is open for a balanced bridge, Thevenizing the circuit, the Thevenin voltage source and its internal resistance v_i' and R' are calculated using Fig. 3.19(e).

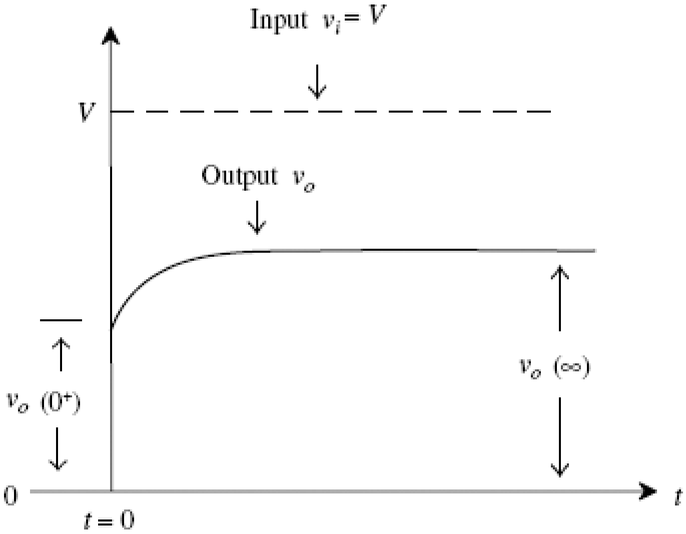


FIGURE 3.19(c) An under-compensated attenuator (C_1 < C_2)

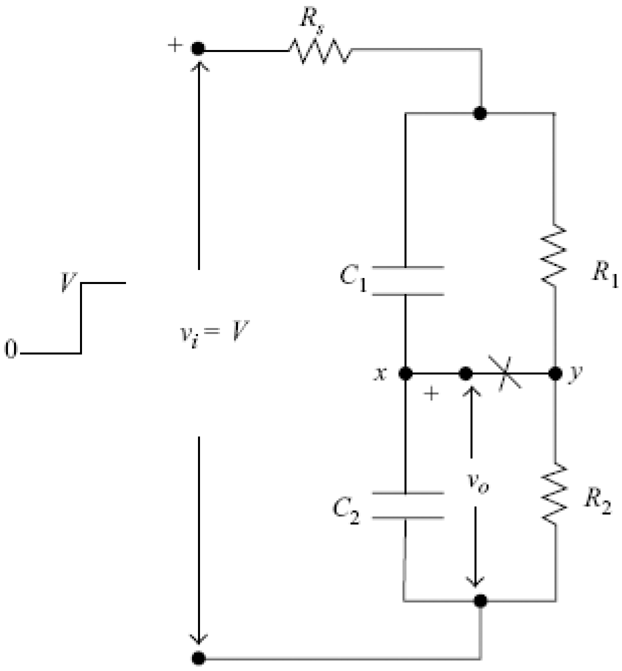


FIGURE 3.19(d) The attenuator taking the source resistance into account

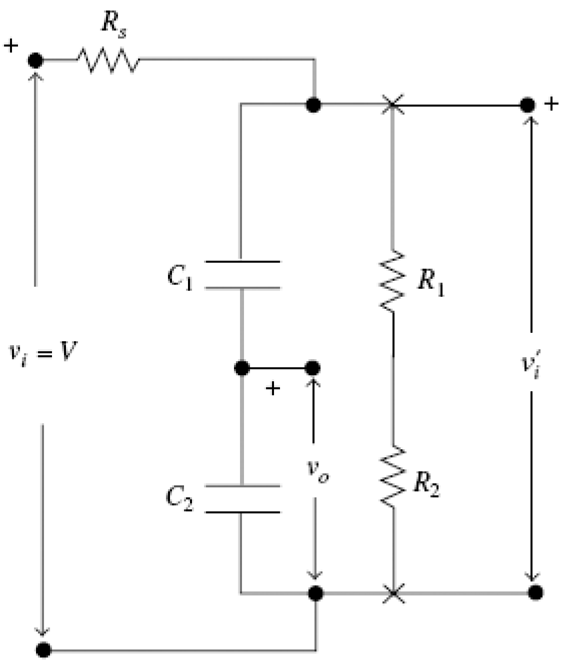


FIGURE 3.19(e) The circuit used to calculate the Thevenin voltage source and its internal resistance

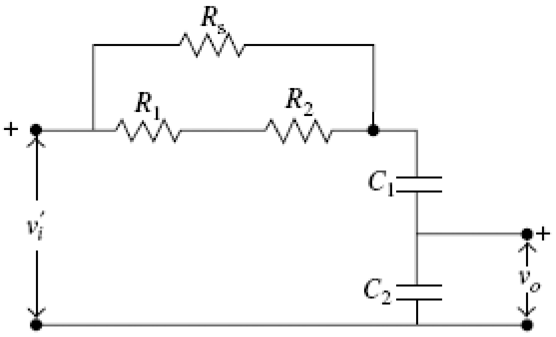


FIGURE 3.19(f) Redrawn circuit of Fig. 3.19(e)

The value of Thevenin voltage source is:

$$v_i' = \frac{v_i(R_1 + R_2)}{R_s + R_1 + R_2}$$

and its internal resistance is:

$$R' = \frac{R_s(R_1 + R_2)}{R_s + R_1 + R_2}$$

The above circuit now reduces to that shown in Fig. 3.19(f). Usually $R_s \lll (R_1 + R_2)$, hence, $R_s \parallel (R_1 + R_2) \approx R_s$. Thus the circuit in Fig. 3.19(f) reduces to that shown in Fig. 3.19(g). This is a low-pass circuit with time constant $\tau_s = R_sC_s$, where C_s is the series combination of C_1 and C_2; $C_s = C_1C_2/(C_1 + C_2)$. The output of the attenuator is an exponential with time constant τ_s; and if τ_s is small, the output almost follows the input. Alternately, consider the situation when a step voltage V from a source having R_s as its internal resistance, is connected to a circuit which has C_2 between its output terminals, [see Fig. 3.19(h)].

This being a low-pass circuit (can also be termed as an uncompensated attenuator), with time constant $\tau(= R_sC_2)$, its output will be an exponential with rise time t_r, where

$$t_r = 2.2\,\tau = 2.2\,R_sC_2. \tag{3.47}$$

Now consider the compensated attenuator, shown in Fig. 3.19(g), where the internal resistance of the source R_s is taken into account. The time constant of this circuit is $\tau_s(= R_sC_s)$ and the rise time τ_r' is:

$\tau_r' = 2.2\ R_sC_s$ where $C_s = C_1C_2/(C_1 + C_2)$

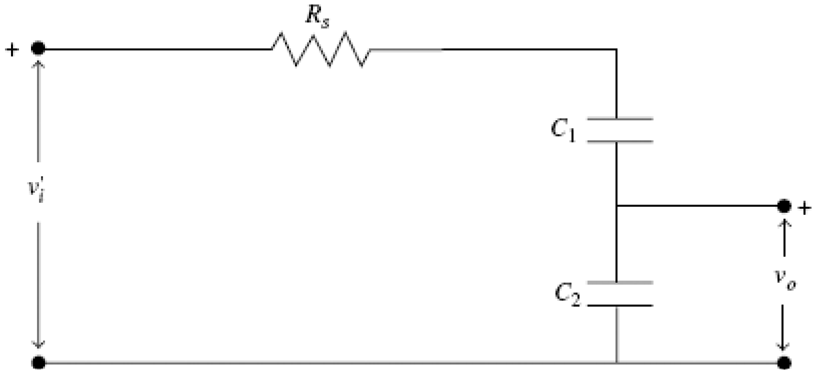


FIGURE 3.19(g) The final reduced circuit of a compensated attenuator

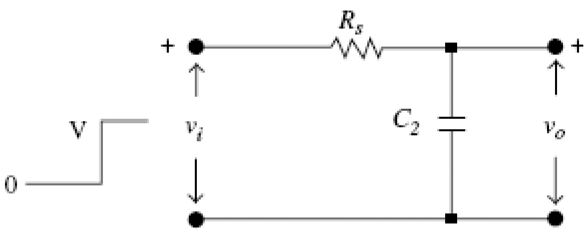


FIGURE 3.19(h) A low-pass circuit (uncompensated attenuator)

$$t_r' = 2.2\,\frac{R_sC_1C_2}{C_1 + C_2} \tag{3.48}$$

From Eqs. (3.47) and (3.48):

$(t_r'/t_r) = [C_1/(C_1 + C_2)] = \alpha$

where α is the attenuation constant, which tells us by what amount the signal is reduced at the output.

$$t_r' = \alpha\, t_r \tag{3.49}$$

If $\alpha = 0.5$:

$$t_r' = 0.5t_r \tag{3.50}$$

From Eq. (3.50), it seen that the output signal from a compensated attenuator has a negligible rise time when compared to the output signal from an un-compensated attenuator. It means that the step voltage, V is more faithfully reproduced at the output of a compensated attenuator, which is its main advantage.

A perfectly compensated attenuator is sometimes used to reduce the signal amplitude when the signal is connected to a CRO to display a waveform. A typical CRO probe may be represented as in Fig. 3.20.Example 3.7 helps to further elucidate and elaborate the functioning of the attenuator circuit.

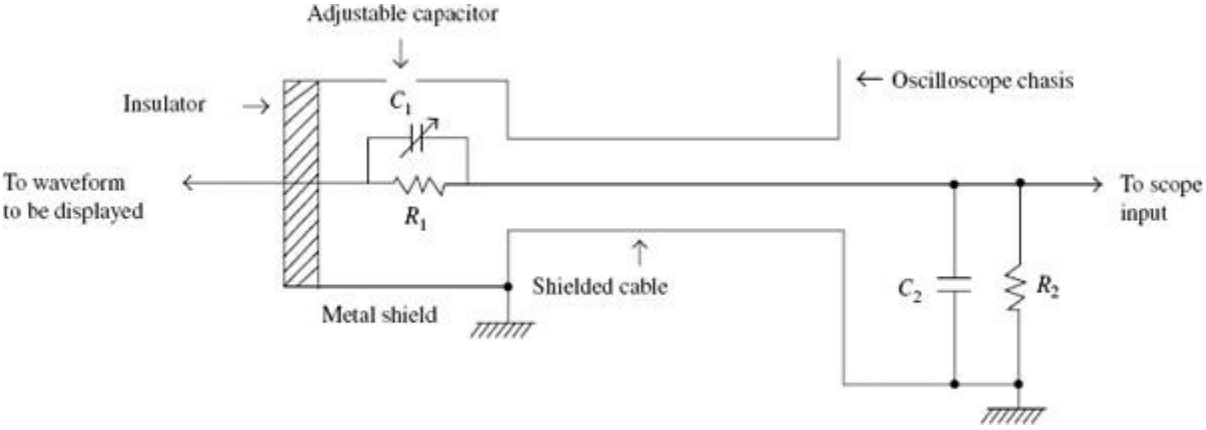


FIGURE 3.20 A typical CRO probe

EXAMPLE

Example 3.7: Calculate the output voltages and draw the waveforms when (a) C_1 = 75 pF, (b) C_1 = 100 pF, (c) C_1 = 50 pF for the circuit shown in Fig. 3.21(a). The input step voltage is 50 V.

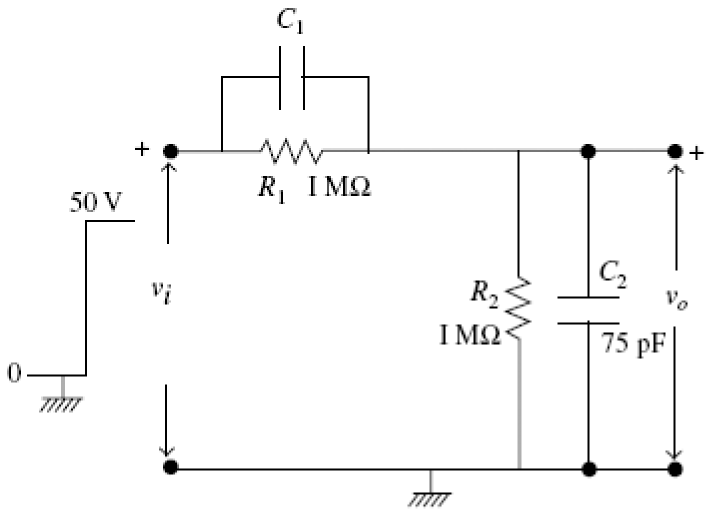


FIGURE 3.21(a) The given attenuator circuit

Solution: For perfect compensation, $R_1C_1 = R_2C_2$. Here $R_1 = R_2$.

1. When $C_1 = 75$ pF, then the attenuator is perfectly compensated. The rise time of the output waveform is zero.

Attenuation, $$\alpha = \frac{R_2}{R_1 + R_2} = \frac{1}{1+1} = 0.5$$

$v_o(0+) = v_o(\infty) = \alpha v_i = 0.5 \times 50 = 25$ V

2. When $C_1 = 100$ pF, then the attenuator is over-compensated, hence $v_o(0^+) > v_o(\infty)$.
The output at $t = 0+$,

$$v_o(0+) = v_i \times \frac{C_1}{C_1 + C_2} = 50 \times \frac{100}{100 + 75} = 28.6 \text{ V}$$

The output at $t = \infty$,

$$v_o(\infty) = v_i \times \frac{R_2}{R_1 + R_2} = 50 \times \frac{1}{1+1} = 25 \text{ V}$$

From Fig. 3.21(b):

$R = \dfrac{R_1 R_2}{R_1 + R_2}$ and $C = C_1 + C_2$

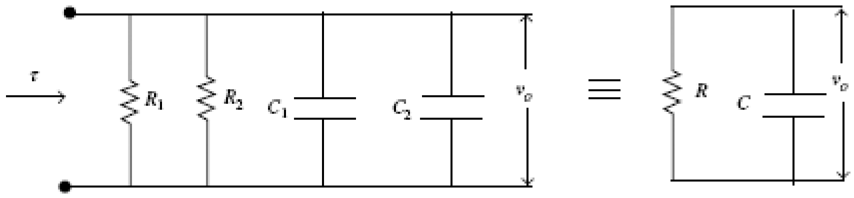


FIGURE 3.21(b) The equivalent circuit to get the time constant for the decay of the overshoot

Time constant τ_1 with which the overshoot at $t = 0^+$ decays to the steady-state value is:

$$\tau_1 = \frac{R_1 R_2}{R_1 + R_2}(C_1 + C_2) = \frac{1 \times 1}{1+1} \times 10^6 \times (100 + 75) \times 10^{-12} = 87.5 \ \mu\text{s}$$

Fall time $t_f = 2.2\ \tau_1 = 2.2 \times 87.5 \times 10^{-6} = 192.5\ \mu$s

3. When $C_1 = 50$ pF, then the attenuator is under-compensated.

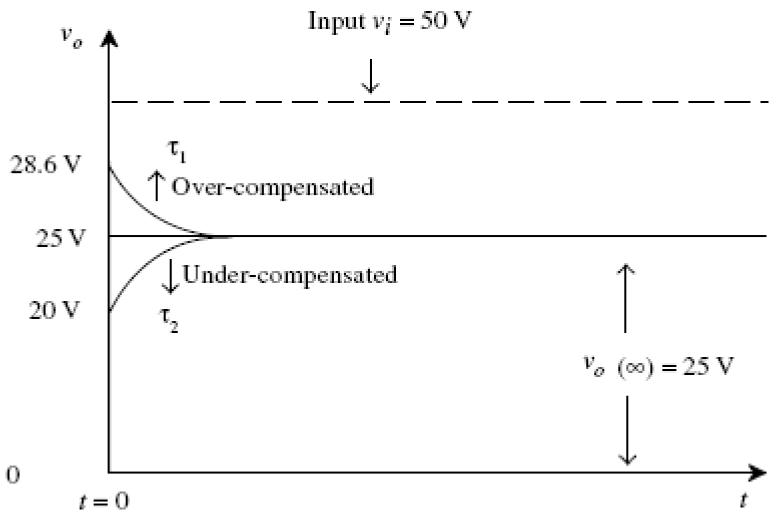


FIGURE 3.21(c) The input and output responses

The output at $t = 0+$:

$$v_o(0^+) = v_i \times \frac{C_1}{C_1 + C_2} = 50 \times \frac{50}{50 + 75} = 20\text{ V}$$

The output at $t = \infty$:

$$v_o(\infty) = v_i \times \frac{R_2}{R_1 + R_2} = 50 \times \frac{1}{1 + 1} = 25\text{ V}$$

The time constant, τ_2, with which the output rises to the steady-state value is:

$$\tau_2 = \frac{R_1 R_2}{R_1 + R_2}(C_1 + C_2) = \frac{1 \times 1}{1 + 1} \times 10^6 \times (50 + 75) \times 10^{-12} = 62.5\ \mu\text{s}$$

Rise time, $t_r = 2.2\ \tau_2$

$t_r = 2.2 \times 62.5 \times 10^{-6} = 137.5\ \mu$s

RLC *CIRCUITS*

RLC circuits behave altogether differently when compared to either *RL* or *RC* circuits. *RLC* circuits are resonant circuits. These can be either series resonant circuits or parallel resonant circuits. A parallel *RLC* circuit is used as a tank circuit in an oscillator to generate oscillations (this is the feedback network that produces the phase shift of 180°). The *RLC* circuit is also used in tuned amplifiers to select a desired frequency band at the output. When a sinusoidal signal is applied as input to a series *RLC* circuit [see Fig. 3.23(a)], the frequency-vs-current characteristic is as shown in Fig. 3.23(b).

At resonance: $X_L = X_C$

$$\omega_o L = 1/\omega_o C$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \qquad \textbf{(3.51)}$$

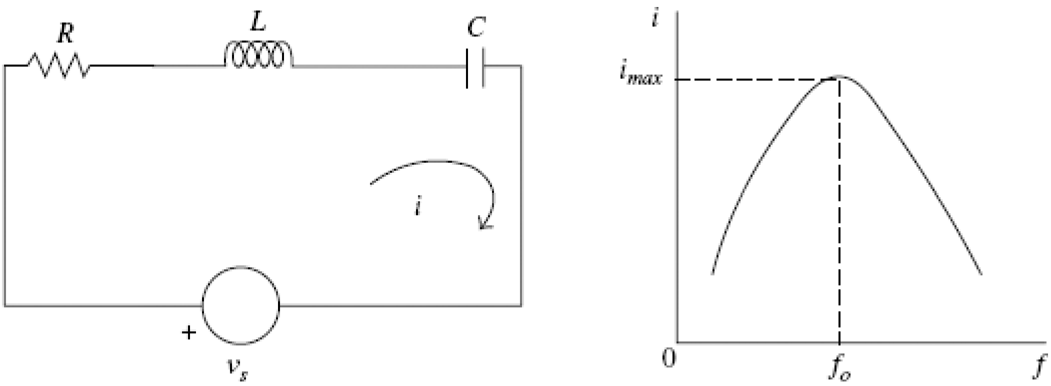


FIGURE 3.23(a) An *RL*C series circuit with sinusoidal input; (b) the frequency-vs-current characteristic

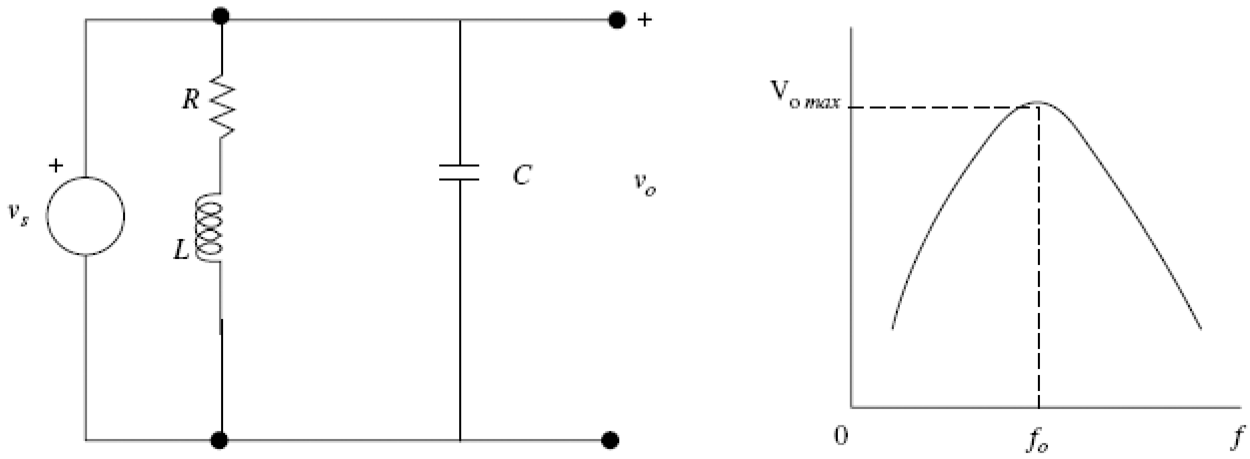


FIGURE 3.23(c) A parallel *RL*C resonant circuit with sinusoidal input; (d) the frequency-vs- voltage characteristic

At resonance, the impedance is minimum, purely resistive and equal to *R*. The current at the resonant frequency, f_o is maximum, termed i_{max}. Let us now consider a parallel resonant circuit [see Fig. 3.23(c)] and its frequency-vs-v_o characteristic, shown in Fig. 3.23(d). In the parallel resonant circuit, the impedance is maximum at resonance and hence, the voltage is maximum at f_o. The figure of merit of a tuned circuit, denoted by *Q*, is given as:

$$Q = \omega_o RC = \frac{(\omega_o L)}{R}$$

The larger the value of *Q*, the sharper the response characteristic of the tuned circuit.

The Response of the *RLC* Parallel Circuit to a Step Input

Consider the *RLC* circuit shown in Fig. 3.24(a). Applying Laplace transforms, the above circuit is redrawn shown in Fig. 3.24(b). The impedance of the parallel combination of *Ls* and 1/*Cs* is:

$$Z_p(s) = \frac{Ls\left(\frac{1}{Cs}\right)}{Ls + \frac{1}{Cs}} \qquad \textbf{(3.52)}$$

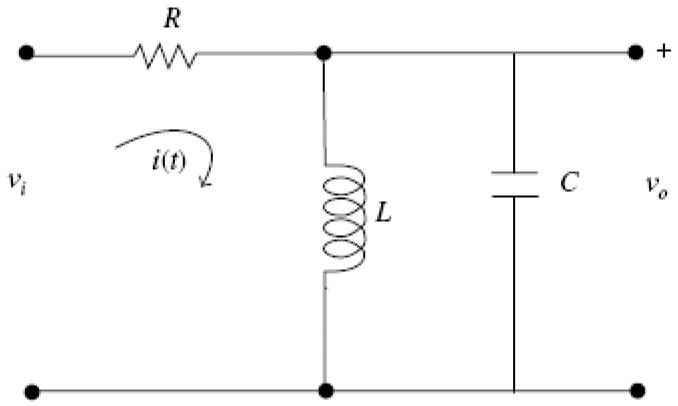


FIGURE 3.24(a) *RLC* parallel circuit

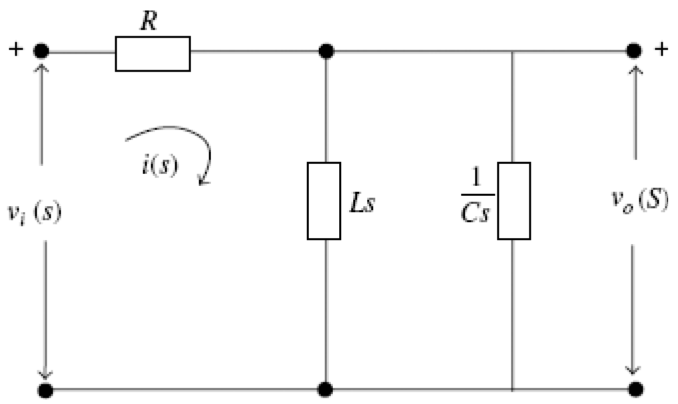


FIGURE 3.24(b) The Laplace circuit of Fig. 3.24(a)

Multiplying the numerator and denominator by *Cs* we get:

$$Z_p(s) = \frac{Ls}{LCs^2+1} \qquad v_o(s) = \frac{v_i(s) \times Z_p(s)}{R + Z_p(s)} = v_i(s)\frac{\dfrac{Ls}{LCs^2+1}}{R + \dfrac{Ls}{LCs^2+1}}$$

$$v_o(s) = v_i(s)\frac{Ls}{RLCs^2 + Ls + R}$$

Therefore,

$$\frac{v_o(s)}{v_i(s)} = \frac{Ls}{RLCs^2 + Ls + R} \qquad \textbf{(3.53)}$$

The characteristic equation is:

$$RLCs^2 + Ls + R = 0 \qquad \textbf{(3.54)}$$

The roots of this characteristic equation are:

$$s_1, s_2 = \frac{-L + \sqrt{L^2 - 4(RLC)R}}{2RLC} = \frac{-L}{2RLC} \pm \sqrt{\frac{L^2}{4R^2L^2C^2} - \frac{4R^2LC}{4R^2L^2C^2}}$$

$$s_1, s_2 = \frac{-1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}} \qquad s_1, s_2 = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}} \qquad \textbf{(3.55)}$$

Let *K*, the damping constant, be given by:

$$K = \frac{1}{2R}\sqrt{\frac{L}{C}} \qquad \textbf{(3.56)}$$

From Eq. (3.51), the resonant frequency of the tank circuit is:

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$T_o = \frac{1}{f_o} = 2\pi\sqrt{LC} \qquad \textbf{(3.57)}$$

From Eqs. (3.56) and (3.57):

$$\frac{K}{T_o} = \frac{1}{2R}\sqrt{\frac{L}{C}} \times \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times 2RC}$$

Therefore,

$$\frac{1}{2RC} = \frac{2\pi K}{T_o} \qquad \textbf{(3.58)}$$

Putting Eq. (3.58) in Eq. (3.55): Therefore,

$$s_1, s_2 = \frac{-2\pi K}{T_o} \pm \sqrt{\left(\frac{2\pi K}{T_o}\right)^2 - \frac{1}{LC}} = \frac{-2\pi K}{T_o} \pm \frac{2\pi K}{T_o} \times \sqrt{1 - \frac{4R^2C^2}{LC}}$$

From Eq. (3.56) we have:

$$\frac{4R^2C}{L} = \frac{1}{K^2}$$

Therefore,

$$s_1, s_2 = \frac{-2\pi K}{T_o} \pm \frac{2\pi K}{T_o} \times \sqrt{1 - \frac{1}{K^2}} = \frac{-2\pi K}{T_o} \pm \frac{2\pi K}{T_o} \times \sqrt{\frac{K^2 - 1}{K^2}}$$

$$s_1, s_2 = \frac{-2\pi K}{T_o} \pm \frac{2\pi K}{T_o K} \times \sqrt{-1(1 - K^2)} = \frac{-2\pi K}{T_o} \pm \frac{j2\pi}{T_o}\sqrt{1 - K^2} \qquad \textbf{(3.59)}$$

From Eq. (3.53):

$$\frac{v_o(s)}{v_i(s)} = \frac{Ls}{RLCs^2 + Ls + R}$$

For unit step voltage as input:

$$\frac{v_o(s)}{\frac{V}{s}} = \frac{Ls}{RLCs^2 + Ls + R} \qquad \frac{V_o(s)}{V} = \frac{L}{RLCs^2 + LS + R}$$

$$\frac{v_o(s)}{V} = \frac{L}{RLC\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)} = \frac{1}{RC}\left[\frac{1}{(s - s_1)(s - s_2)}\right]$$

Applying partial fractions:

$$\frac{v_o(s)}{V} = \left[\frac{A}{(s - s_1)} + \frac{B}{(s - s_2)}\right] = \frac{1}{RC}\left[\frac{1}{(s - s_1)(s - s_2)}\right] \qquad \frac{1}{RC} = A(s - s_2) + B(s - s_1)$$

Putting $s = s_1$

$$\frac{1}{RC} = A(s_1 - s_2) \qquad A = \frac{1}{RC(s_1 - s_2)}$$

Putting $s = s_2$

$$\frac{1}{RC} = -B(s_1 - s_2) \qquad B = \frac{-1}{RC(s_1 - s_2)}$$

$$\frac{v_o(s)}{V} = \frac{1}{RC(s_1 - s_2)}\left[\frac{1}{(s - s_1)} - \frac{1}{(s - s_2)}\right]$$

Applying inverse Laplace transform to both sides, we get:

$$\frac{v_o(t)}{V} = \frac{1}{RC(s_1 - s_2)}\left(e^{s_1 t} - e^{s_2 t}\right) \quad \textbf{(3.60)}$$

From Eq. (3.59):

(i) If $K = 0$:

$$s_1, s_2 = \pm\frac{j2\pi}{T_o} \quad \textbf{(3.61)}$$

$$s_1 - s_2 = \frac{j2\pi}{T_o} - \left(-\frac{j2\pi}{T_o}\right) = \frac{j4\pi}{T_o} = \frac{2j2\pi}{T_o} \quad \textbf{(3.62)}$$

Using Eq. (3.60):

$$\frac{v_o(t)}{V} = \frac{1}{RC\dfrac{2\pi}{T_o}}\frac{\left(e^{(j2\pi/T_o)t} - e^{-(j2\pi/T_o)t}\right)}{2j}$$

$$\frac{v_o(t)}{V} = \frac{1}{\dfrac{2\pi RC}{T_o}}\left[\sin\left(\frac{2\pi}{T_o}\times t\right)\right] \quad \textbf{(3.63)}$$

Let

$$x = \frac{t}{T_o} \quad \textbf{(3.64)}$$

Therefore,

$$\frac{v_o(t)}{V} = \frac{1}{\dfrac{2\pi RC}{T_o}}\left[\sin(2\pi x)\right] \quad \textbf{(3.65)}$$

If we substitute the value of T_o from Eq. (3.57) in Eq. (3.65):

$$\frac{v_o(t)}{V} = \frac{2\pi\sqrt{LC}}{2RC\pi}\left[\sin(2\pi x)\right] = \frac{1}{R}\times\sqrt{\frac{L}{C}}\times\sin(2\pi x)$$

$$\frac{v_o(t)}{V} = 2K\sin(2\pi x) \quad \textbf{(3.66)}$$

Thus, $K \rightarrow 0$, as $R \rightarrow \infty$. Here, K can not be zero as assumed ideally, since $R = \infty$ means open-circuiting the resistance R in the circuit shown in Fig. 3.24(a), which is absurd because the excitation is not connected to the circuit when $R = \infty$. However, R can be made very large, in which case K becomes very small, though not zero as expected. The output has a smaller amplitude but is oscillatory in nature, as seen from Eq. (3.66). Thus, when a step is applied as input to the RLC circuit in Fig. 3.24(a), with $K = 0$ (practically very small), the response is un-damped oscillations, as shown in Fig. 3.24(c).

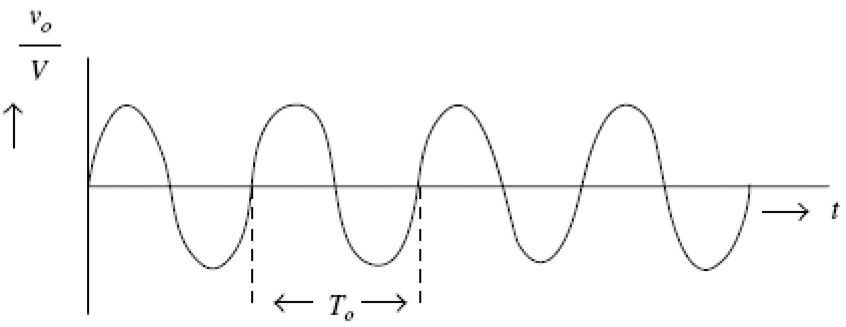


FIGURE 3.24(c) The response to $K = 0$

(ii) If $K < 1$, it is a case of under-damping as shown in Fig. 3.24(d). For this condition, from Eq. (3.59):

$$s_1, s_2 = \frac{-2\pi K}{T_o} \pm \frac{j2\pi}{T_o}\sqrt{1-K^2}$$

$$s_1 = \frac{-2\pi K}{T_o} + \frac{j2\pi}{T_o}\sqrt{1-K^2} \qquad s_2 = \frac{-2\pi K}{T_o} - \frac{j2\pi}{T_o}\sqrt{1-K^2}$$

Therefore,

$$s_1 - s_2 = \frac{-2\pi K}{T_o} + \frac{j2\pi}{T_o}\sqrt{1-K^2} + \frac{2\pi K}{T_o} + \frac{j2\pi}{T_o}\sqrt{1-K^2}$$

$$s_1 - s_2 = \frac{j4\pi}{T_o}\sqrt{1-K^2} \tag{3.67}$$

Multiply and divide Eq. (3.67) by K and substitute $K/T_o = 1/4\pi RC$ in it. The resultant equation is:

$$\frac{1}{RC(s_1 - s_2)} = \frac{K}{j\sqrt{1-K^2}}$$

From Eq. (3.60):

$$\frac{v_o(t)}{V} = \frac{K}{j\sqrt{1-K^2}} \times \left(e^{(a+jb)t} - e^{(a-jb)t}\right)$$

where

$$a = \frac{-2\pi K}{T_o} \quad \text{and} \quad b = \frac{2\pi}{T_o}\sqrt{1-K^2}$$

$$\frac{v_o(t)}{V} = \frac{K}{j\sqrt{1-K^2}} \times e^{at}\left(e^{jbt} - e^{-jbt}\right) = \frac{K}{j\sqrt{1-K^2}} \times e^{at} 2j \sin bt$$

$$\frac{v_o(t)}{V} = \frac{2K}{\sqrt{1-K^2}} \times e^{at} \sin bt = \frac{2K}{\sqrt{1-K^2}} \times e^{-2\pi Kt/T_o} \times \sin\left(\frac{2\pi t}{T_o}\sqrt{1-K^2}\right)$$

$$\frac{v_o(t)}{V} = \frac{2K}{\sqrt{1-K^2}} \times e^{-2\pi K x} \times \sin\left(2\pi x\sqrt{1-K^2}\right) \tag{3.68}$$

The output response is an under-damped sinusoidal waveform. The oscillations die down after a few cycles, as shown in Fig. 3.24(d).

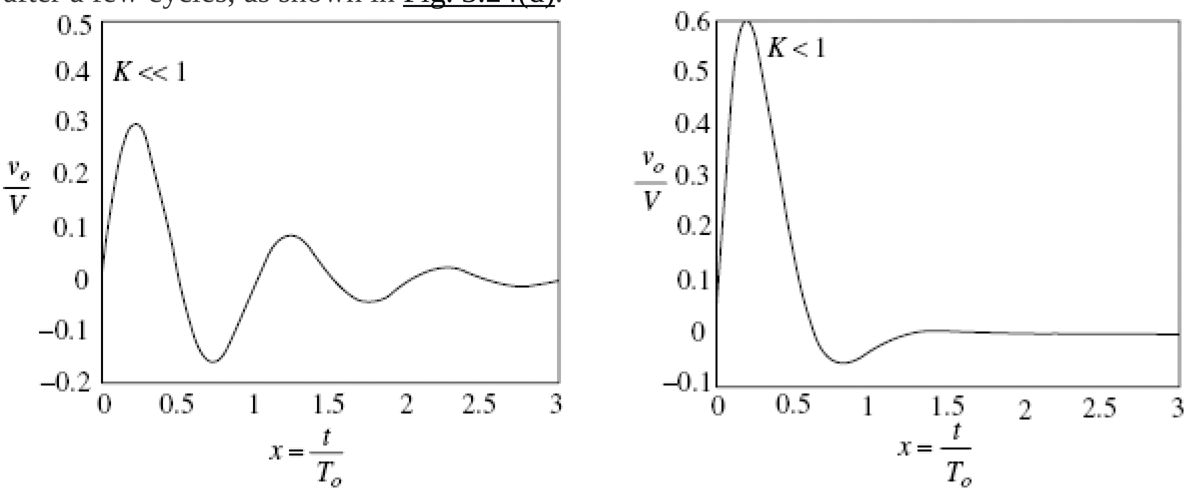


FIGURE 3.24(d) The response to $K < 1$

(iii) If $K = 1$, it is a case of critical damping. If we substitute the K value in the Eq. (3.59), then the roots are $s_1 = s_2 = -2\pi/T_o$. The roots are equal and real.

If the input is a step voltage:

$$\frac{v_o(s)}{V} = \frac{1}{RC(s - s_1)(s - s_2)}$$

Here,

$$s_1 = s_2 = \frac{-2\pi}{T_o}$$

Therefore,

$$\frac{v_o(s)}{V} = \frac{1}{RC(s - s_1)^2}$$

Applying inverse Laplace transform on both sides:

$$\frac{v_o(t)}{V} = \frac{1}{RC} \times te^{s_1 t} = \frac{1}{RC} \times te^{(-2\pi/T_o)\times t} = \frac{1}{RC} \times te^{-2\pi x}$$

where $x = t/T_o$:

$$\frac{v_o(t)}{V} = \frac{4\pi}{4\pi RC} \times te^{-2\pi x}$$

Here:

$$\frac{T_o}{K} = 4\pi RC \qquad \frac{v_o(t)}{V} = \frac{4\pi t}{\frac{T_o}{K}} \times e^{-2\pi x} = \frac{4\pi Kt}{T_o} \times e^{-2\pi x}$$

$$\frac{v_o(t)}{V} = 4\pi Kxe^{-2\pi x} \qquad \textbf{(3.69)}$$

$$\frac{v_o(t)}{V} = 4\pi x e^{-2\pi x} \quad \text{since} \quad K = 1 \tag{3.70}$$

The output response is shown in Fig. 3.24(e).

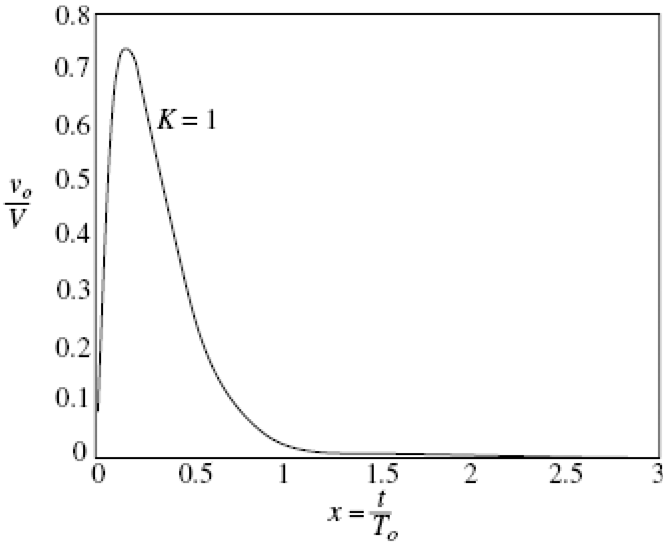


FIGURE 3.24(e) The response to *K* = 1

(iv) If *K* > 1, it is a case of over-damping. If *K* > 1, then the roots, from Eq. (3.59), are:

The Response of the *RLC* Series Circuit to a Step Input

Consider a series *RL*C circuit, shown in Fig. 3.25(a). Applying Laplace transforms, the circuit in Fig. 3.25(a) can be redrawn as shown in Fig. 3.25(b). The total impedance *Z(s*) in this circuit is given by the relation:

$$Z(s) = R + Ls + \frac{1}{Cs}$$

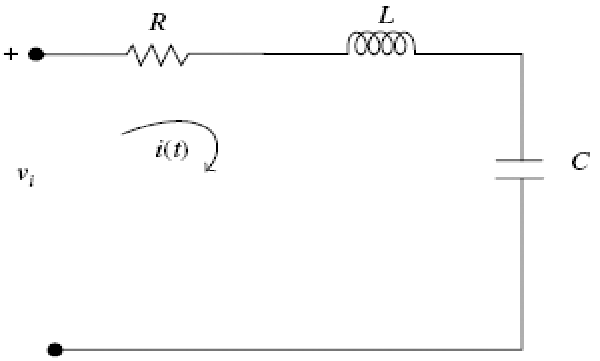


FIGURE 3.25(a) An *RL*C series circuit

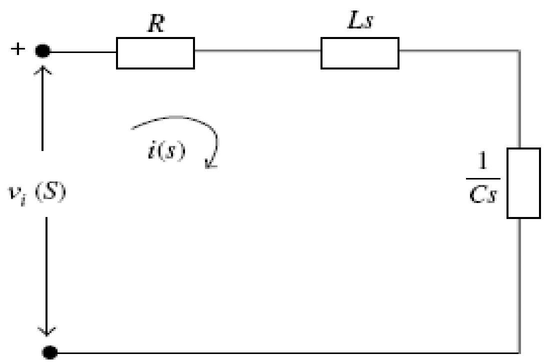


FIGURE 3.25(b) The Laplace circuit of Fig. 3.25(a)

Therefore,

$$i(s) = \frac{v_i(s)}{Z(s)} = \frac{v_i(s)}{R + Ls + \frac{1}{Cs}} \tag{3.75}$$

$$v_i(s) = i(s)\left(R + Ls + \frac{1}{Cs}\right) = \frac{i(s)}{Cs}\left(LCs^2 + RCs + 1\right) \tag{3.76}$$

But

$$v_o(s) = \frac{i(s)}{Cs} \tag{3.77}$$

Substituting Eq. (3.77) in Eq. (3.76):

$$v_i(s) = v_o(s)\left(LCs^2 + RCs + 1\right)$$

$$v_o(s) = \frac{v_i(s)}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} \tag{3.78}$$

For a step input *V*, from Eq. 3.75

$$i(s) = \frac{\frac{V}{s}}{R + Ls + \frac{1}{Cs}} = \frac{V}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} \tag{3.79}$$

The characteristic equation is:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \tag{3.80}$$

The roots of this characteristic equation are:

$$s_1, s_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \tag{3.81}$$

1. If either $(R/2L)^2 > 1/LC$ or $R > 2\sqrt{L/C}$, then both the roots are real and different, the circuit is over-damped.
2. If either $(R/2L)^2 = 1/LC$ or $R = 2\sqrt{L/C}$, then both the roots are real and equal, the circuit is critically damped.
3. If either $(R/2L)^2 < 1/LC$ or $R < 2\sqrt{L/C}$, then both the roots are complex conjugate to each other; the circuit is under-damped.